Day 7: Intro to Inequalities and Quadratic Equations

GED

CLUES
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Student question from PLATO:

Teresa has two brothers, Paul and Steve. Paul and Steve are the same height.

Paul is 16 inches shorter than 1 ½ times Teresa's height.

Steve is 6 inches shorter than 1 1/3 times Teresa's height.

How tall is Teresa?

$$p = \frac{3}{2}\,t\; - 16$$

$$s = \frac{4}{3}t - 6$$

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How tall is Teresa?

p = Paul's height
s = Steve's height
t = Teresa's height

p and s - how are they related?

$$p = \frac{3}{2} t - 16$$

p = s

$$s = \frac{4}{3} t - 6$$

= 3/3
have one third extra
/3 + 1/3 = 4/3

converted a mixed fraction
nto an "improper fraction"
ecause it's easier to do math

$$p = \frac{3}{2}\ t\ - 16 \qquad\qquad s = \frac{4}{3}t - 6$$

and, p=s

$$\frac{3}{2}\ t\ - 16 \quad = \quad \frac{4}{3}t - 6$$

$$p = \frac{3}{2}t - 16 \qquad s = \frac{4}{3}t - 6$$

and, p=s

$$\frac{3}{2}t - 16 = \frac{4}{3}t - 6$$

How many variables do I have? One: (t)
Do I see any t^2 or t^3? No.
So, this is a linear equation with one variable.

Get the variable by itself by doing the same thing to both sides of the equation.

$$\frac{3}{2}t - 16 = \frac{4}{3}t - 6$$

$$\frac{1}{6}t - 16 = -6$$

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$$\frac{3}{2}t - 16 = \cancel{\frac{4}{3}t} - 6$$

$$-\frac{4}{3}t \qquad \cancel{-\frac{4}{3}t}$$

3/2 is one, plus one half extra

4/3 is one, plus one third extra

Which one is bigger?

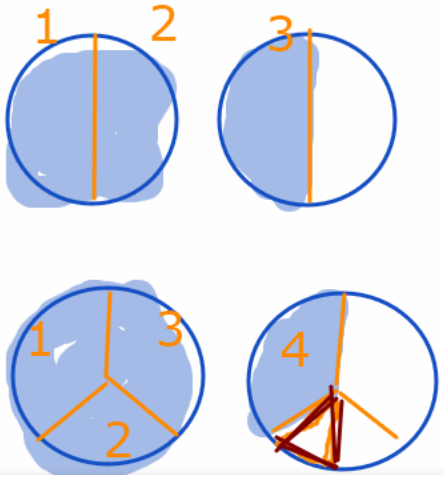
A common denominator between 2 and 3 is 6

$$\frac{3 \cdot 3}{2 \cdot 3} - \frac{4 \cdot 2}{3 \cdot 2}$$

*3 3 / *3 2 - 4 *2 / 3 *2

$$\frac{9}{6} - \frac{8}{6} = \frac{1}{6}t$$

$$\frac{1}{6}t - 16 = 6$$



= 3/2

= 4/3

Inequalities

Learn the Skill

$$\frac{1}{6}t - 16 = -6$$

$$\frac{1}{6}t = 10$$

$$t = 60$$

$$\frac{1}{6}t - 16 = -6$$

$$+16 \qquad +16$$

$$\frac{1}{6}t = 10$$

$$*6 \qquad *6$$

$$t = 60$$

Teresa is 60 inches tall. (This is 5 feet tall, or maybe 1.5, 1.6 meters tall)

Inequalities

1 Learn the Skill

- An **inequality** states that two algebraic expressions are not equal. Inequalities are written with less than ($<$) and greaten than symbols ($>$), as well as the $\geq$ symbol which means "greater than or equal to" and the $\leq$ which means "less than or equal to".

1 Learn the Skill

- An **inequality** states that two algebraic expressior are not equal. Inequalities are written with less th (<) and greaten than symbols (>), as well as th symbol which means "greater than or equal to" ar the ≤ which means "less than or equal to".

Water will be frozen at $\leq$ 0 degrees Celsius

At zero degrees Celsius, water is frozen

Water will be liquid at > 0 degrees Celsius

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1

Learn the Skill

- A solution to an inequality can include an infinite amount of numbers. For example, solutions to $b < 5$ include $b = 4.5, 4, 3.99, 3, 2, 1, 0, -3, -10$, and so on.
- When each individual solution is plotted as a point on a number line, a solid line is formed, which represents the solution set.

Solving Inequalities

Symbol	Words	Example
>	greater than	$x + 3 > 2$
<	less than	$7x < 28$
≥	greater than or equal to	$5 \geq x - 1$
≤	less than or equal to	$2y + 1 \leq 7$

Solving

Our aim is to have x (or whatever the variable is) **on its own** on the left of the inequality sign:

Something like: $x < 5$

or: $y \geq 11$

We call that "solved".

Example: $x + 2 > 12$

Subtract 2 from both sides:

$$x + 2 - 2 > 12 - 2$$

Simplify:

$$x > 10$$

Solved!

How to Solve

Solving inequalities is very like solving equations ... we do most of the same things ...

... but we must also pay attention to the **direction of the inequality**.

Direction: Which way the arrow "points"

Some things can **change the direction**!

$<$ becomes $>$

$>$ becomes $<$

$\leq$ becomes $\geq$

$\geq$ becomes $\leq$

Safe Things To Do

These things **do not affect** the direction of the inequality:

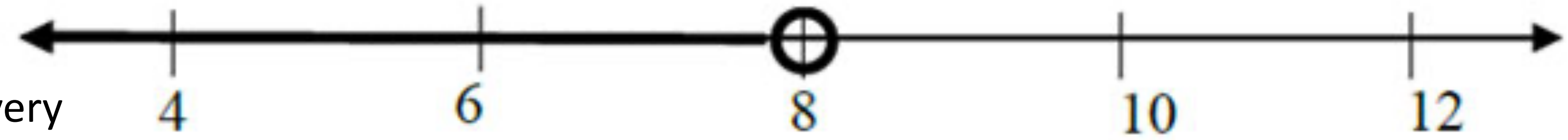
- Add (or subtract) a number from both sides
- Multiply (or divide) both sides by a **positive** number
- Simplify a side

Example: $3x < 7+3$

We can simplify 7+3 without affecting the inequality:

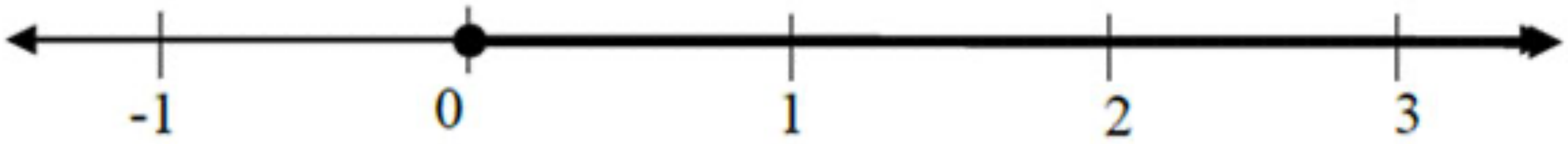
$$3x < 10$$

Graphing Inequalities



For this inequality, every number less than 8 is in the solution set. Draw an open circle at 8 because 8 is not in the solution set. Then draw a solid arrow to the left from 8.

$$x < 8$$



$$x \geq 0$$

For this inequality, each number to the right of 0 as well as 0 is included in the solution set. Draw a closed circle at 0 to show that 0 is included. Then draw a solid arrow pointing to the right

- Five times a number is less than or equal to two times the number plus nine. What is the solution to the inequality?

a) $x \geq 9$
b) $x \leq 9$
c) $x \geq 3$
d) $x \leq 3$

- Five times a number is less than or equal to two times the number plus nine. What is the solution to the inequality?

a) $x \geq 9$

b) $x \leq 9$

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$5x \boxed{\leq} 2x + 9$

When there is a problem written out in words, always just write the equation first.

$$5x \leq 2x + 9$$

$$-2x \qquad -2x$$

$$3x \leq 9$$

$$/3 \quad /3$$

$$x \leq 3$$

Solving an inequality is the same as solving an equation. Do the same thing to both sides, to get x by itself.

Practice!

- What is the solution to the inequality $x + 5 > 4$

a) $x > 1$
b) $x < -1$
c) $x < 1$
d) $x > -1$

Practice!

- Kara has \$15 and Brett \$22. Together, they have less than the amount needed to buy a pair of concert tickets. Which inequality describes their situation?

a) $37 < x$
b) $x + 15 < 25$
c) $x \leq 37$
d) $x + 22 \leq 15$

Quadratic Equations

1 Learn the Skill

- **Quadratic equations** are equations set in the form:

$$ax^2 + bx + c = 0$$

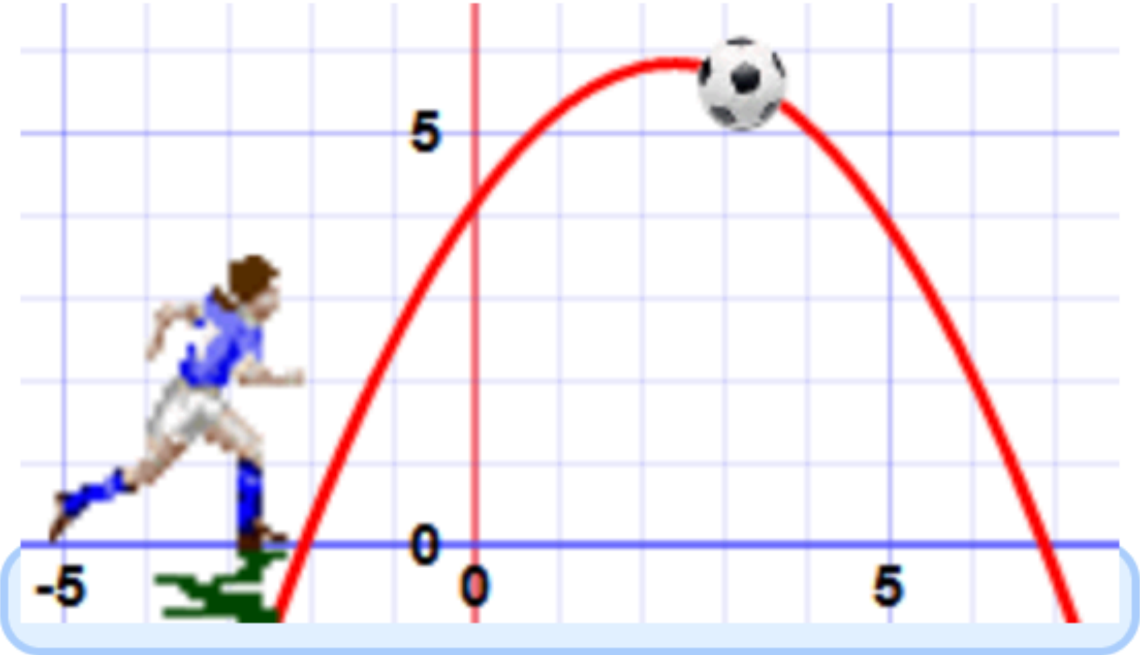
(**a**, **b**, and **c** can have any value, except that **a** can't be 0.)

An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Quadratic Equations make nice curves, like this one:



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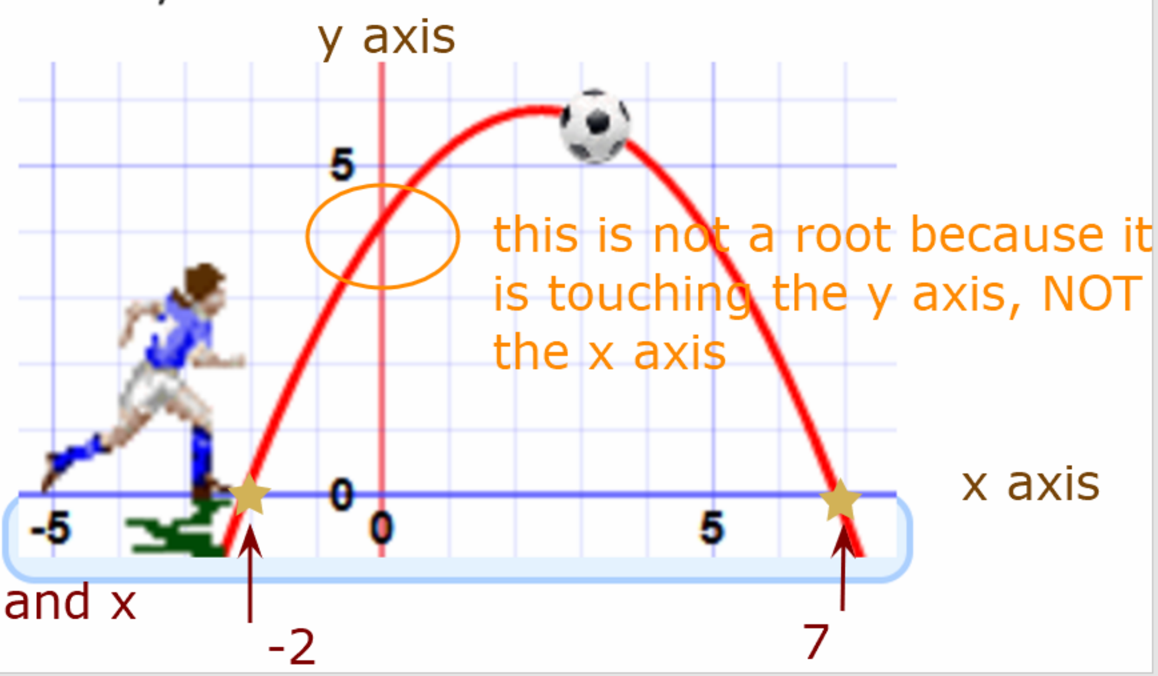


In a quadratic equation, you will often be asked to find the ROOTS

The root is the value of x which makes the equation equal to zero. It is where the curve touches the x - axis on a graph.

In this example, x = -2 and x = 7

Name

The name **Quadratic** comes from "quad" meaning square, because the variable gets squared (like $\mathbf{x^2}$).

It is also called an "Equation of Degree 2" (because of the "2" on the **x**)

Here are some examples:

$2x^2 + 5x + 3 = 0$		In this one **a=2**, **b=5** and **c=3**
$x^2 - 3x = 0$		This one is a little more tricky: • Where is **a**? Well **a=1**, as we don't usually write "$1x^2$" • **b = −3** • And where is **c**? Well **c=0**, so is not shown.
$5x - 3 = 0$		**Oops!** This one is **not** a quadratic equation: it is missing x^2 (in other words **a=0**, which means it can't be quadratic)

Hidden Quadratic Equations!

As we saw before, the **Standard Form** of a Quadratic Equation is

$$ax^2 + bx + c = 0$$

But sometimes a quadratic equation doesn't look like that!

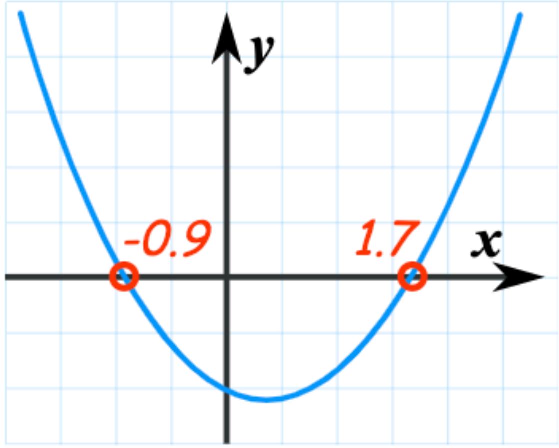
For example:

In disguise		In Standard Form	a, b and c
$\mathbf{x^2 = 3x - 1}$	Move all terms to left hand side	$\mathbf{x^2 - 3x + 1 = 0}$	a=1, b=−3, c=1
$\mathbf{2(w^2 - 2w) = 5}$	Expand (undo the brackets), and move 5 to left	$\mathbf{2w^2 - 4w - 5 = 0}$	a=2, b=−4, c=−5
$\mathbf{z(z-1) = 3}$	Expand, and move 3 to left	$\mathbf{z^2 - z - 3 = 0}$	a=1, b=−1, c=−3

How To Solve Them?

The "**solutions**" to the Quadratic Equation are where it is **equal to zero**.

They are also called "**roots**", or sometimes "**zeros**"



There are usually 2 solutions (as shown in this graph).

And there are a few different ways to find the solutions:

We can Factor the Quadratic (find what to multiply to make the Quadratic Equation)

Or we can Complete the Square

Or we can use the special **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Just plug in the values of a, b and c, and do the calculations.

We will look at this method in more detail now.

To factor a Quadratic is to:

find what to multiply to get the Quadratic

It is called "Factoring" because we find the factors (a factor is something we multiply by)

Example:

Multiplying **(x+4)** and **(x−1)** together (called Expanding) gets $\mathbf{x^2 + 3x - 4}$:

$$(x+4)(x-1) \underset{\text{Factor}}{\overset{\text{Expand}}{\rightleftarrows}} x^2 + 3x - 4$$

So **(x+4)** and **(x−1)** are factors of $\mathbf{x^2 + 3x - 4}$

Just to be sure, let us check:

$$\begin{aligned}(x+4)(x-1) &= x(x-1) + 4(x-1)\\ &= x^2 - x + 4x - 4\\ &= x^2 + 3x - 4 \checkmark\end{aligned}$$

Yes, **(x+4)** and **(x−1)** are definitely factors of $\mathbf{x^2 + 3x - 4}$

Did you see that Expanding and Factoring are opposites?

$$(x+4)(x-1) \underset{\text{Factor}}{\overset{\text{Expand}}{\rightleftarrows}} x^2 + 3x - 4$$

? x ? = 

It is like trying to find which ingredients went into a cake to make it so delicious.

It can be hard to figure out!

Common Factor

First check if there any common factors.

Example: what are the factors of $6x^2 - 2x = 0$?

6 and **2** have a common factor of **2**:

$$2(3x^2 - x) = 0$$

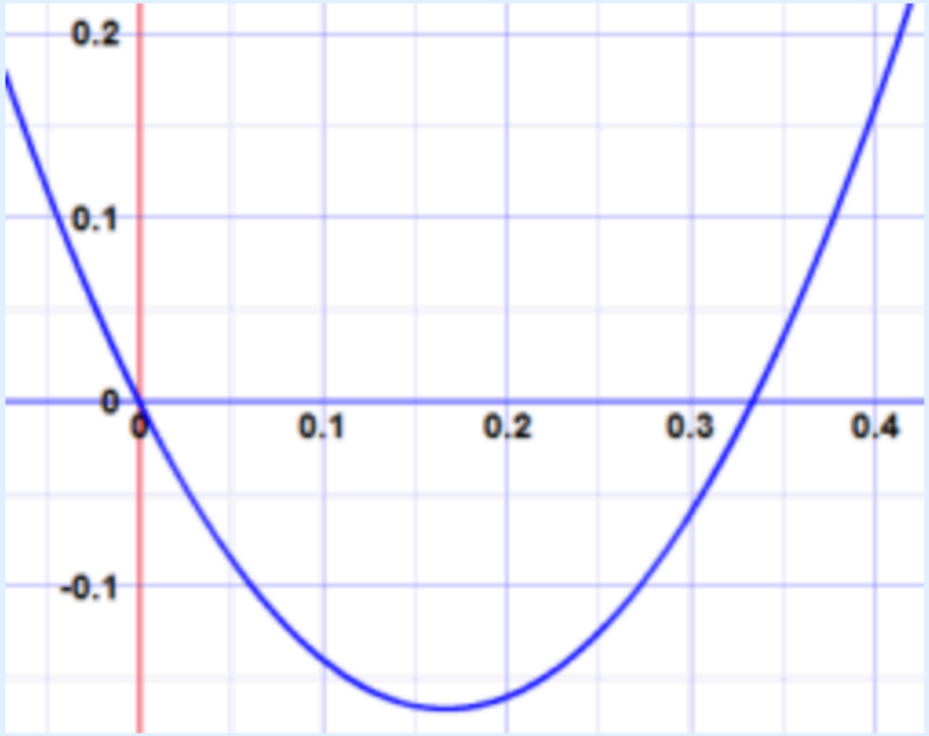
And $\mathbf{x^2}$ and $\mathbf{x}$ have a common factor of $\mathbf{x}$:

$$2x(3x - 1) = 0$$

And we have done it! The factors are **2x** and **3x − 1**, We can now also find the **roots** (where it equals zero):

- $2x$ is 0 when $\mathbf{x = 0}$
- $3x - 1$ is zero when $\mathbf{x = \frac{1}{3}}$

And this is the graph (see how it is zero at $x=0$ and $x=\frac{1}{3}$):



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- 3x − 1 is zero when $\mathbf{x = \frac{1}{3}}$

And this is the graph (see how it is zero at x=0 and x=$\frac{1}{3}$):

6x^2 - 2x = 0

$\boxed{2x}\boxed{(3x -1)}$ = 0

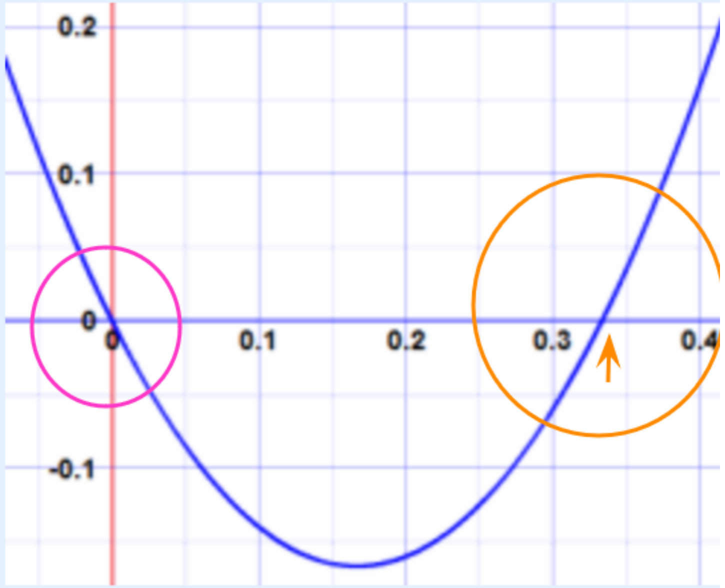
2x = 0 OR 3x - 1 = 0

/2 /2 +1 +1

x = 0 3x = 1

/3 /3

x = 1/3



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A Method For Simple Cases

Luckily there is a method that works in simple cases.

With the quadratic equation in this form:

$$ax^2 + bx + c = 0$$

Step 1: Find two numbers that multiply to give ac (in other words a times c), and add to give b.

Example: $2x^2 + 7x + 3$

ac is $2 \times 3 =$ **6** and b is **7**

So we want two numbers that multiply together to make 6, and add up to 7

In fact **6** and **1** do that ($6 \times 1 = 6$, and $6 + 1 = 7$)

How do we find 6 and 1?

It helps to list the factors of ac=**6**, and then try adding some to get b=**7**.

Factors of 6 include 1, 2, 3 and 6.

Aha! 1 and 6 add to 7, and 6×1=6.

Step 2: Rewrite the middle with those numbers:

Rewrite 7x with **6**x and **1**x:

$$2x^2 + \mathbf{6x + x} + 3$$

Step 3: Factor the first two and last two terms separately:

The first two terms $2x^2 + 6x$ factor into $2x(x+3)$

The last two terms $x+3$ don't actually change in this case

So we get:

$$2x(x+3) + (x+3)$$

Step 4: If we've done this correctly, our two new terms should have a clearly visible common factor.

In this case we can see that $(x+3)$ is common to both terms, so we can go:

Start with:	$2x(x+3) + (x+3)$
Which is:	$2x(x+3) + 1(x+3)$
And so:	$\mathbf{(2x+1)(x+3)}$

Done!

Check: $(2x+1)(x+3) = 2x^2 + 6x + x + 3 = \mathbf{2x^2 + 7x + 3}$ (Yes)

Let's see Steps 1 to 4 again, in one go:

$$\mathbf{2x^2 + 7x + 3}$$

$$2x^2 + 6x + x + 3$$

$$2x(x+3) + (x+3)$$

$$2x(x+3) + 1(x+3)$$

$$\mathbf{(2x+1)(x+3)}$$

Factors: Algebraic Identities

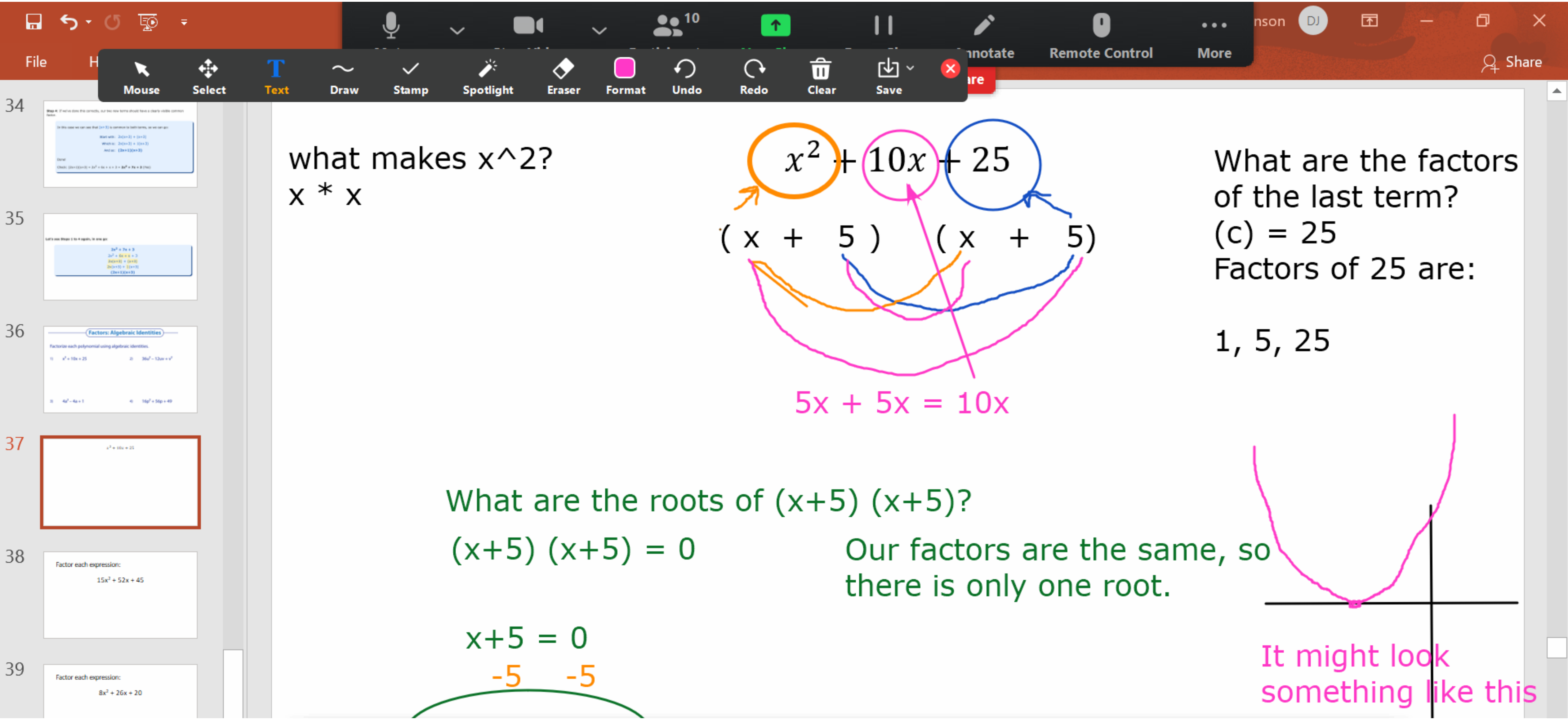
Factorize each polynomial using algebraic identities.

1) $x^2 + 10x + 25$

2) $36u^2 - 12uv + v^2$

3) $4a^2 - 4a + 1$

4) $16p^2 + 56p + 49$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Undo
Redo
Clear
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Remote Control
More
Share
what makes x^2?
x * x
$x^2 + 10x + 25$
(x + 5) (x + 5)
5x + 5x = 10x
What are the factors of the last term?
(c) = 25
Factors of 25 are:
1, 5, 25
What are the roots of (x+5) (x+5)?
(x+5) (x+5) = 0
Our factors are the same, so there is only one root.
x+5 = 0
-5 -5
It might look something like this

x = - 5

to make 4a^2
I can use 2a * 2a or
4a * a

$4a^2 - 4a + 1$

(2a - 1) (2a - 1)

To
1 *

-2a - 2a =

4a, a adds to 5
2a and 2a adds to 4

(2a - 1) (2a - 1) = 0

2a - 1 = 0
+1 +1

2a = 1
/2 /2

a = 1/2 or 0.5

Once again, our two factors are the same, so we only have to solve once.

To make 1, I can use either 1*1 OR -1 * -1

-2a – 2a = -4a

Factor each expression:

$$15x^2 + 52x + 45$$

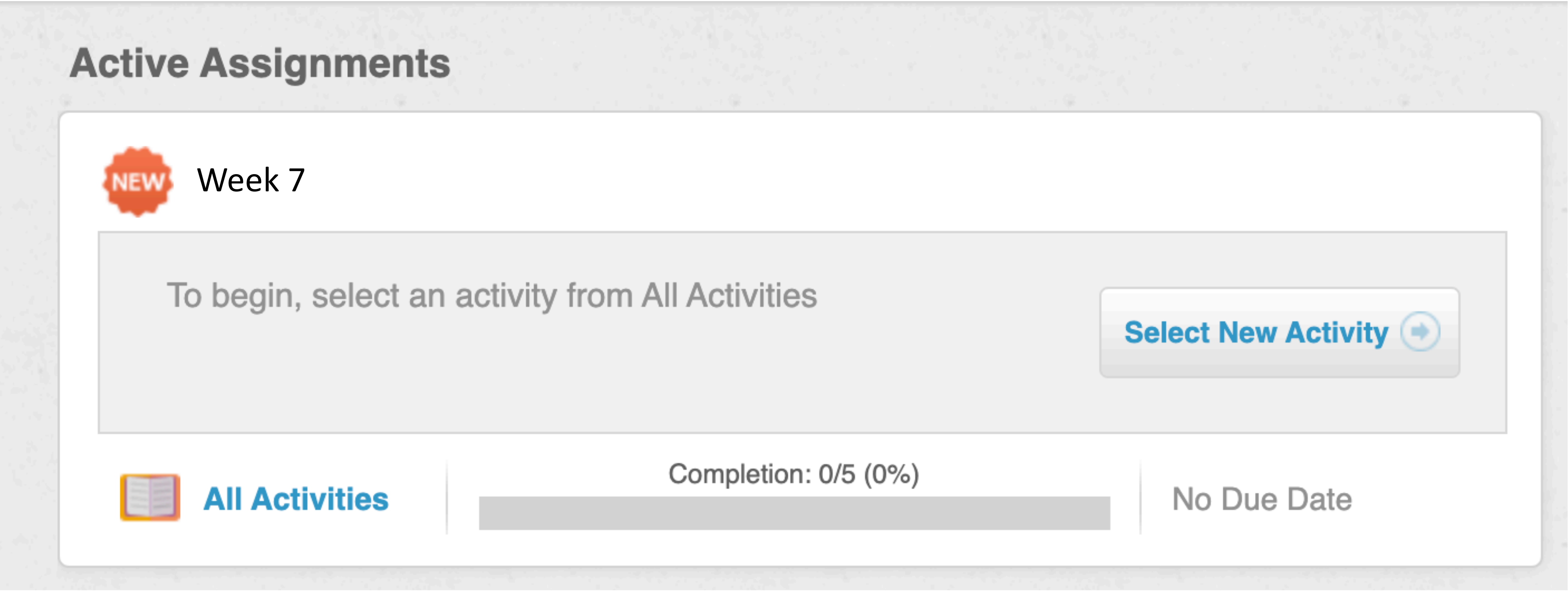
Factor each expression:

$$8x^2 + 26x + 20$$

Factor each expression:

$$21x^2 - x - 2$$

Homework!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NEW
Week 7
To begin, select an activity from All Activities
Select New Activity
All Activities
Completion: 0/5 (0%)
No Due Date