Day 6:
More Factoring and Linear Equations

- Each letter stands for a different variable.
- Where there are no signs (+, -, /, *), we assume things are multiplied
  - So, “2x” means two times x

I can combine variables which are the same, but not variables which are different.

2a + a = 3a

“Xy” means “x times y”

Can I simplify this?

2x + xy

- x^2 + x + 1 + $3x^2$

- $4x^2$ + x + 1

Factoring in Algebra

Numbers have factors:

$$\underset{\text{Factor}}{2} \times \underset{\text{Factor}}{3} = 6$$

And expressions (like $\mathbf{x^2+4x+3}$) also have factors:

$$\underbrace{(x+3)}_{\text{Factor}}\underbrace{(x+1)}_{\text{Factor}} = x^2 + 4x + 3$$

Factoring: Finding what to multiply together to get an expression.

It is like "splitting" an expression into a multiplication of simpler expressions.

Example: factor $2y+6$

Both 2y and 6 have a common factor of 2:

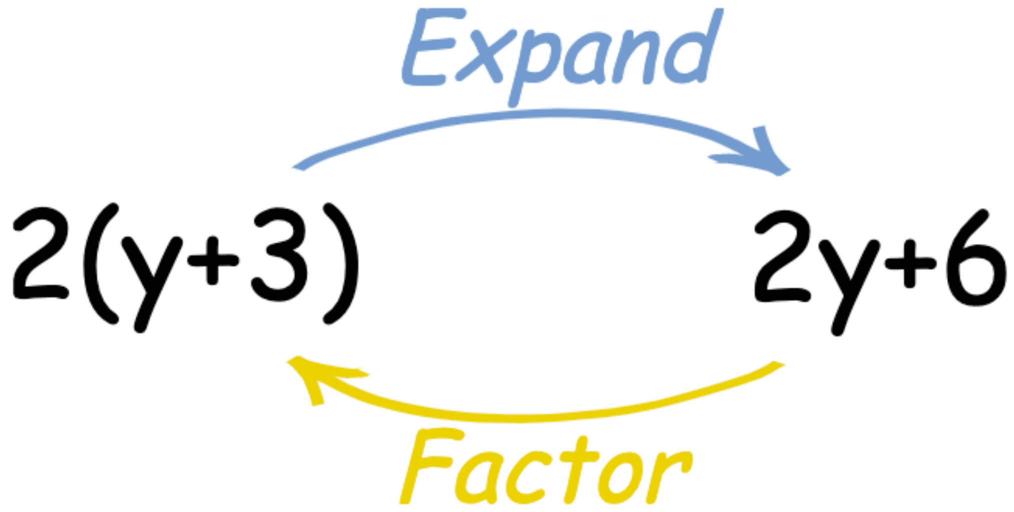
- $2y$ is $2 \times y$
- 6 is 2×3

So we can factor the whole expression into:

$$2y+6 = 2(y+3)$$

So **2y+6** has been "factored into" **2** and **y+3**

Factoring is also the opposite of Expanding:



Common Factor

In the previous example we saw that 2y and 6 had a common factor of **2**

But to do the job properly we need the **highest common factor**, including any variables

Example: factor $3y^2+12y$

Firstly, 3 and 12 have a common factor of 3.

So we could have:

$$3y^2+12y = 3(y^2+4y)$$

But we can do better!

$3y^2$ and $12y$ also share the variable y.

Together that makes $3y$:

- $3y^2$ is $3y \times y$
- $12y$ is $3y \times 4$

So we can factor the whole expression into:

$$3y^2+12y = 3y(y+4)$$

Check: **$3y(y+4) = 3y \times y + 3y \times 4 = 3y^2+12y$**

Factor each binomial

1) $2ab + 4c$

2) $t^3 - ts$

3) $3n - 6m$

4) $5y^2 + y$

Factor each binomial

$\cancel{t} * t * t$

common factor in each term

1) $\dfrac{\cancel{2}ab}{\cancel{2}} + \dfrac{4c}{2}$

ab + 2c

2) $\dfrac{t^3}{\cancel{t}} - \dfrac{\cancel{t}s}{\cancel{t}}$

t ^ 2 - s

$\dfrac{5 * \cancel{y} * y}{\cancel{y}}$

y / y = 1

3) $\dfrac{\cancel{3}n}{\cancel{3}} - \dfrac{6m}{3}$

n - 2m

4) $\dfrac{5\cancel{y}^2}{\cancel{y}} + \dfrac{\cancel{y}}{\cancel{y}}$

5y + 1

More Complicated Factoring

Factoring Can Be Hard !

The examples have been simple so far, but factoring **can** be very tricky.

Because we have to figure **what got multiplied** to produce the expression we are given!

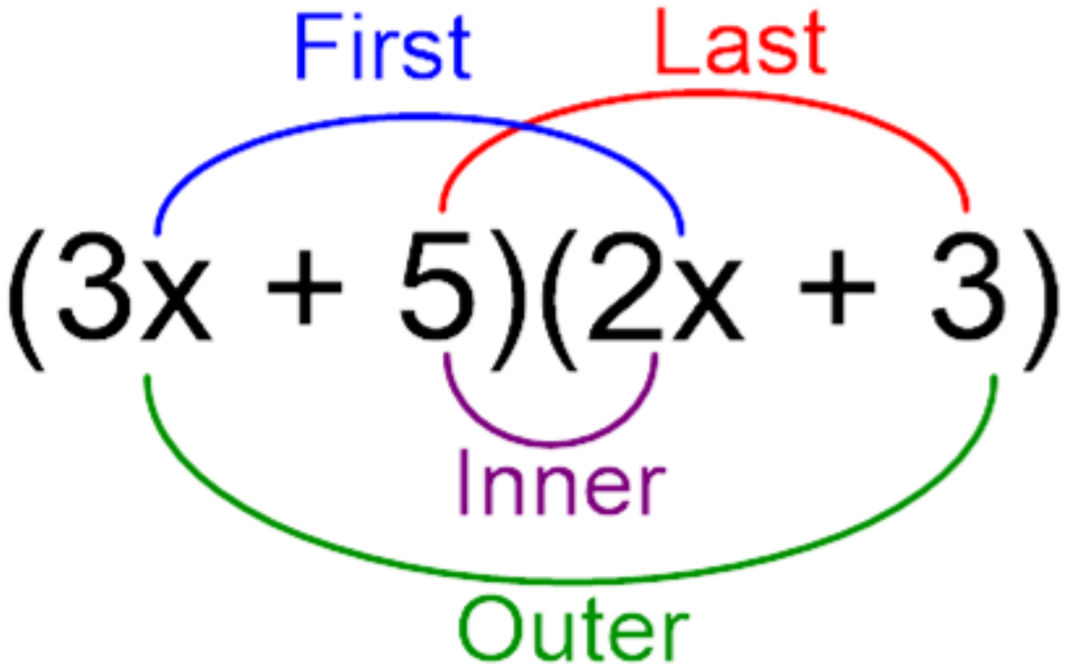
? x ? = 

It is like trying to find which ingredients went into a cake to make it so delicious.

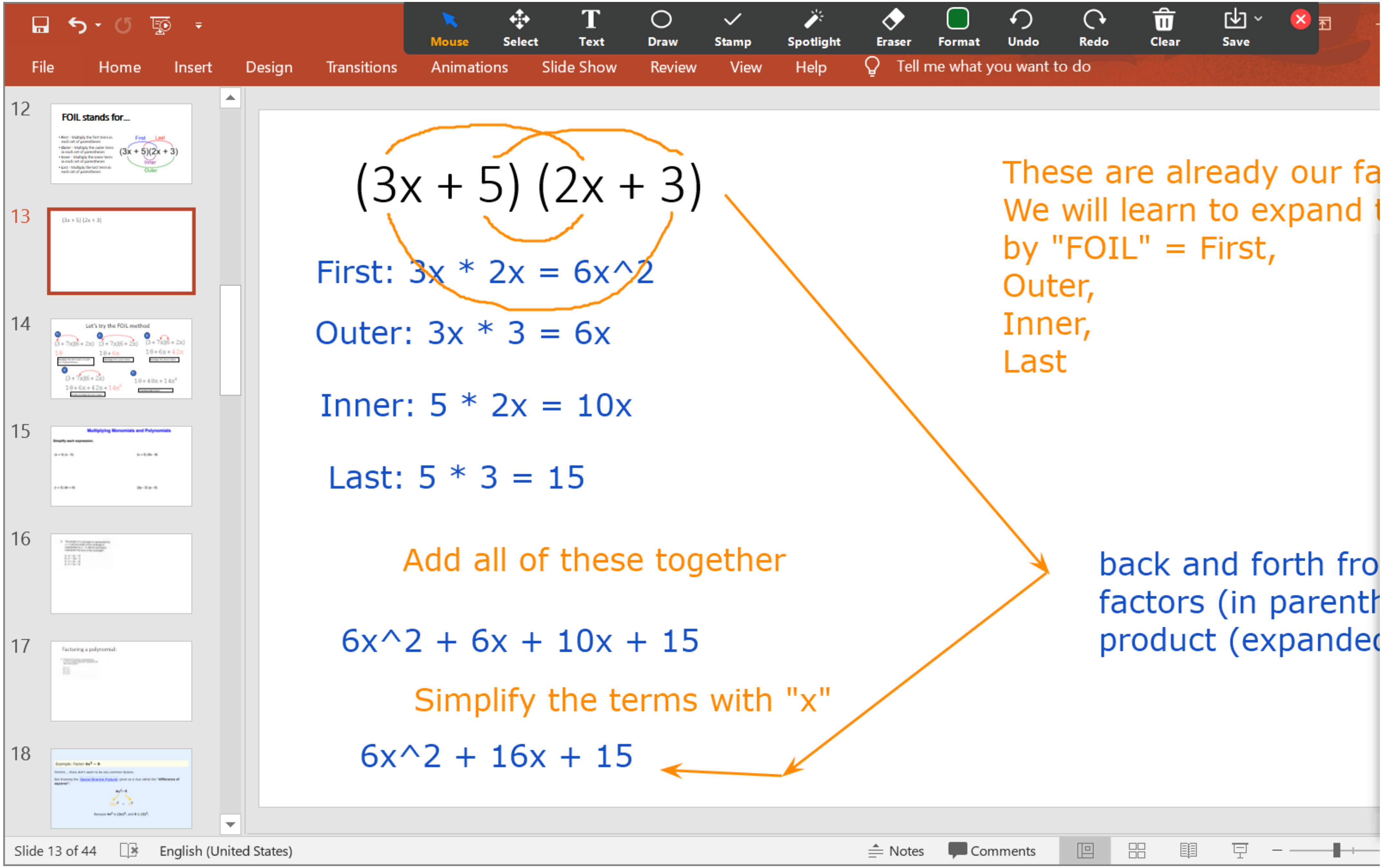
It can be hard to figure out!

FOIL stands for…

- **F**irst - Multiply the first term in each set of parentheses
- **O**uter - Multiply the outer term in each set of parentheses
- **I**nner - Multiply the inner term in each set of parentheses
- **L**ast - Multiply the last term in each set of parentheses



$(3x + 5)(2x + 3)$


(3x + 5) (2x + 3)
First: 3x * 2x = 6x^2
Outer: 3x * 3 = 6x
Inner: 5 * 2x = 10x
Last: 5 * 3 = 15
Add all of these together
6x^2 + 6x + 10x + 15
Simplify the terms with "x"
6x^2 + 16x + 15
These are already our fa
We will learn to expand t
by "FOIL" = First,
Outer,
Inner,
Last
back and forth fro
factors (in parenth
product (expanded
Slide 13 of 44
English (United States)

Let's try the FOIL method

1

$$(3+7x)(6+2x)$$

$$18$$

Multiply the first term of each set of parentheses

2

$$(3+7x)(6+2x)$$

$$18+6x$$

Multiply the outer terms

3

$$(3+7x)(6+2x)$$

$$18+6x+42x$$

Multiply the inner terms

4

$$(3+7x)(6+2x)$$

$$18+6x+42x+14x^2$$

Finally multiply the last terms

5

$$18+48x+14x^2$$

Combine like terms

Multiplying Monomials and Polynomials

Simplify each expression.

$(z + 4)(z - 5)$

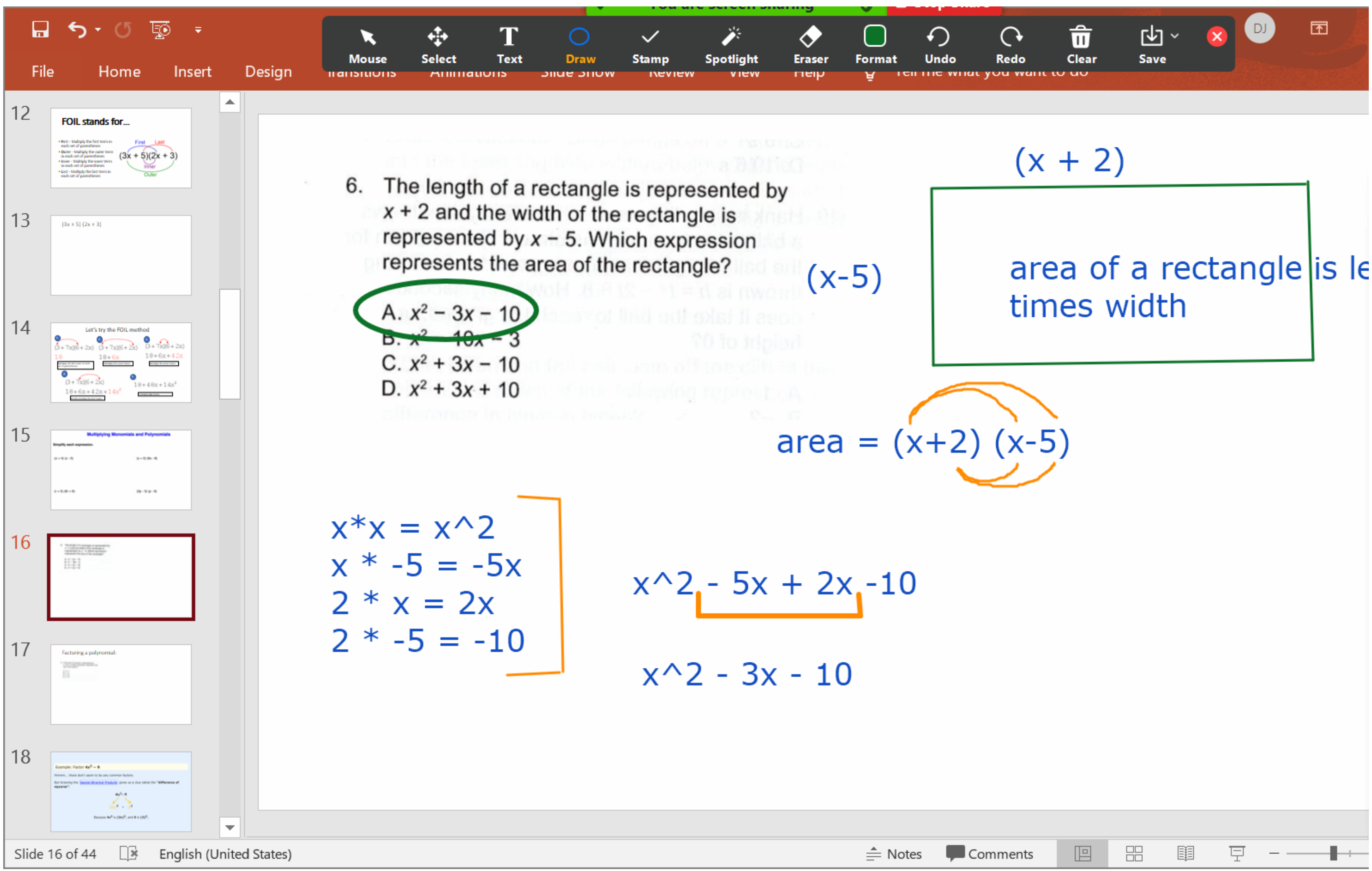
$(x + 5)(6x - 8)$

$(r + 5)(6r + 9)$

$(2p - 3)(p - 6)$

6. The length of a rectangle is represented by $x + 2$ and the width of the rectangle is represented by $x - 5$. Which expression represents the area of the rectangle?

A. $x^2 - 3x - 10$
B. $x^2 - 10x - 3$
C. $x^2 + 3x - 10$
D. $x^2 + 3x + 10$


6. The length of a rectangle is represented by x + 2 and the width of the rectangle is represented by x − 5. Which expression represents the area of the rectangle?
A. x² − 3x − 10
B. x² − 10x − 3
C. x² + 3x − 10
D. x² + 3x + 10
(x + 2)
(x-5)
area of a rectangle is le
times width
area = (x+2) (x-5)
x*x = x^2
x * -5 = -5x
2 * x = 2x
2 * -5 = -10
x^2 - 5x + 2x -10
x^2 - 3x - 10
Slide 16 of 44
English (United States)
Notes
Comments

Factoring a polynomial:

11. If the area of a square is represented by $x^2 + 6x + 9$, which expression represents one side of the square?

A. $x + 1$
B. $x + 3$
C. $x + 6$
D. $x + 9$

Example: Factor $\mathbf{4x^2 - 9}$

Hmmm... there don't seem to be any common factors.

But knowing the Special Binomial Products gives us a clue called the **"difference of squares"**:

$$4x^2 - 9$$

$$(2x)^2 - 3^2$$

Because $\mathbf{4x^2}$ is $\mathbf{(2x)^2}$, and $\mathbf{9}$ is $\mathbf{(3)^2}$, So we have:

$$4x^2 - 9 = (2x)^2 - (3)^2$$

And that can be produced by the difference of squares formula:

$$(a+b)(a-b) = a^2 - b^2$$

Where **a** is 2x, and **b** is 3.

So let us try doing that:

$$(2x+3)(2x-3) = (2x)^2 - (3)^2 = 4x^2 - 9$$

Yes!

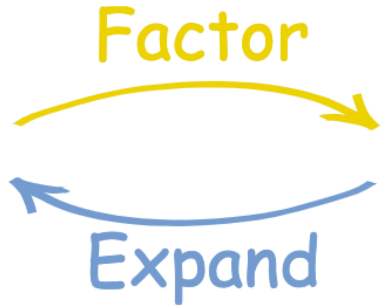
How can you learn to do that? By getting lots of practice, and knowing "Identities"!

So the factors of $\mathbf{4x^2 - 9}$ are $\mathbf{(2x+3)}$ and $\mathbf{(2x-3)}$:

$$\text{Answer: } 4x^2 - 9 = (2x+3)(2x-3)$$

How can you learn to do that? By getting lots of practice and knowing "Identities"!

- Here is a list of common "Identities" (including the **"difference of squares"** used above).
- It is worth remembering these, as they can make factoring easier.



$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$

$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3+3a^2b+3ab^2+b^3 = (a+b)^3$$

$$a^3-3a^2b+3ab^2-b^3 = (a-b)^3$$

Advice

The factored form is usually best.

When trying to factor, follow these steps:

- "Factor out" any common terms
- See if it fits any of the identities, plus any more you may know
- Keep going till you can't factor any more

Factors: Algebraic Identities

Factorize each polynomial using algebraic identities.

1) $x^2 + 10x + 25$

2) $36u^2 - 12uv + v^2$

3) $4a^2 - 4a + 1$

4) $16p^2 + 56p + 49$

One-Variable Linear Equations

1

Learn the Skill

- A **one-variable linear equation** is an equation that consists of expressions involving only number values and products of constants and a variable, such as

- $2x + 6 = 12$.

- The solution of a one-variable linear equation is the value of the variable that makes the equation true.

1

$$2x + 6 = 12.$$
$$\quad -6 \qquad -6$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$$

$$x = 3$$

$$2\,(3) + 6 =$$
$$6 + 6 = 12$$

What value does x have?
SOLVE the equation for x

Get x by itself.

I can always perform sa[me]
operation on both sides o[f]
equation, and the equati[on]
still be true.

1

Learn the Skill

- To solve a one-variable linear equation, use inverse operations to group variable terms on one side of the equation and constant terms on the other side of the equation. For example: $2x + 6 = 12$
- Subtract 6 from each side of the equal (=) sign so that $2x=6$.

1 Learn the Skill

- Next, use **inverse operations** to isolate the variable. Inverse operations are operations that undo each other. Addition and subtraction are inverse operations, as are multiplication and division.
- For example, with 2x + 6 = 12, both 2x and 6 can be divided by 2, so that x = 3.

2

Practice the Skill

Solve the equation

$5x + 7 = 19 - 3x$		
$5x + 3x + 7 = 19 - 3x + 3x$	←	Add 3x to both sides
$8x + 7 = 19$	←	Group like terms
$8x + 7 - 7 = 19 - 7$	←	Subtract 7 from both sides
$8x = 12$	←	Group like terms
$8x/8 = 12/8$	←	Divide both sides by 8
$x = 1.5$	←	Simplify

1. What value of x makes the equation $3x - 9 = 6$ true?

a. -1
b. -3
c. 5
d. 15

<u>USING LOGIC</u>

An equation states that two expressions are equal. When working with an equation, you must perform the same operations in the same order to both sides of the equation!

2. Solve the equation for x.

$0.5x - 4 = 12$

a. 4
b. 8
c. 16
d. 32

3. What value of *y* makes the equation true?

$$5y + 6 = 3y - 14$$

a. -1
b. -2.5
c. -4
d. -10

4. Each month, Cameron earns \$1,200 in salary plus an 8% commission in sales. The equation $T = 1{,}200 + 0.08s$ represents Cameron's total earnings each month. In July, Cameron earned a total, T, of \$2,800. What was the value of Cameron's sales in July?

a. \$15,000
b. \$16,000
c. \$20,000
d. \$50,000

4. Each month, Cameron earns $1,200 in salary plus an 8% commission in sales. The equation T = 1,200 + 0.08s represents Cameron's total earnings each month. In July, Cameron earned a total, T, of $2,800. What was the value of Cameron's sales in July?

$$T = 1{,}200 + 0.08s$$

$$2{,}800 = 1{,}200 + \underline{0.08s}$$

????

$$-1{,}200 \qquad -1{,}200$$

$$\frac{1{,}600}{0.08} = \frac{0.08s}{0.08}$$

$$20{,}000 = s$$

a. $15,000

b. $16,000

c. $20,000

d. $50,000

In July, he earned $1,200 in salary and made $20,000 of sales, which was worth $1,600 in commissions. His total earnings was $2,800.

5. A rectangular yard is x feet wide. The yard is 4 feet longer than it is wide. The perimeter P of the yard is given by the equation $P = 4x + 8$. If the perimeter of the yard is 84 feet, how long is the yard?

a. 19 feet

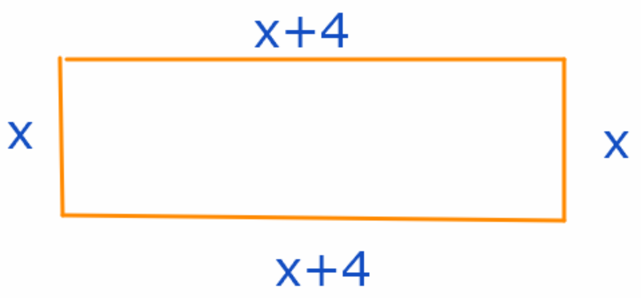
b. 23 feet

c. 24 feet

d. 28 feet

The perimeter is the distance all the way around.

5. A rectangular yard is x feet wide. The yard is 4 feet longer than it is wide. The perimeter P of the yard is given by the equation P = 4x + 8. If the perimeter of the yard is 84 feet, how long is the yard?



$$4x + 8 = P$$
$$4x + 8 = 84$$
$$-8 \qquad -8$$
$$\frac{4x}{4} = \frac{76}{4}$$
$$x = 19$$

a. 19 feet
b. 23 feet
c. 24 feet
d. 28 feet

Two-Variable Linear Equations

1 Learn the Skill

- A **two-variable linear equation** is mathematical sentence that equates two expressions, such as

$$4x + 2y = 14$$

- whose terms are made up of number values and products of constants and variables.
- Only one variable, such as x or y, but not both, may be present in a single term. A system of two-variable linear equations often may be solved using either:

1

Learn the Skill

- the **substitution method**, in which one of the equations is solved for one variable, and the value of that variable is substituted into the original equation to solve for the second variable; or

1

Learn the Skill

- the **linear combination method**, or elimination, in which one or both equations are multiplied by a constant to produce new coefficients that are opposites, so that one variable may be cancelled out and the resulting equation may be solved for the other variable.

Solve the set of two-variable linear equations:

$$\begin{cases} 2x + y = 7 \\ 4x - 2y = 2 \end{cases}$$

Substitution Method

a Solve the first equation for y:

$$2x - 2x + y = 7 - 2x$$
$$y = 7 - 2x$$

Substitute 7 – 2x for y; solve for x:

$$4x - (7 - 2x) = 2$$
$$4x - 14 + 4x = 2$$
$$8x - 14 + 14 = 2 + 14$$
$$8x = 16$$
$$x = 2$$

c Substitute x = 2 and solve for y:

$$2(2) + y = 7$$
$$4 + y = 7$$
$$y = 3$$

The solution is (2,3)

Linear Combination Method

b Multiply the first equation by 2:

$$4x + 2y = 14$$

Add to the second equation:

$$\begin{array}{l} 4x + 2y = 14 \\ \underline{4x - 2y = 2} \\ 8x = 16 \\ x = 2 \end{array}$$

c Substitute x = 2 and solve for y:

$$2(2) + y = 7$$
$$4 + y = 7$$
$$y = 3$$

The solution is (2,3)

Note and Tips

a. Either equation can be solved for either variable. It is easiest to solve for the variable with a coefficient of 1 or -1.

b. This system also could be solved by multiplying the first equation by -2, adding it to the second equation, and solving the new equation for y.

c. The value of the variable can be substituted into either of the original equations. You will get the same value for the second variable.

<u>TEST-TAKING TIPS</u>

The solution of a linear equation reflects the values of the variable that make both equations true. Check that your solution is corrected by substituting, ordered-pair values into the two original equations.

1. Which ordered pair is the solution of the system of linear equations?

$$x + 3y = 1$$
$$2x + 2y = 6$$

a. (-2, 1)
b. (3, 1)
c. (4, -1)
d. (7, -2)

1. Which ordered pair is the solution of the system of linear equations?

$$x + 3y = 1$$
$$2x + 2y = 6$$

Step 1: get x by itself
Step 2: substitute (now, your equation has only variable, which is y
Step 3: solve for (y)

a. (-2, 1)
b. (3, 1)
c. (4, -1)
d. (7, -2)

"solving by Substitution"

1. Which ordered pair is the solution of the system of linear equations?

$x + 3y = 1$

$2x + 2y = 6$

Both of the equat
true. (x and y are
same in both equ
What are x and y

$x + 3y = 1$

$-3y \qquad -3y$

$x + 0 = 1 - 3y$

$x = 1 - 3y$

$2x + 2y = 6$

$2(1 - 3y) + 2y = 6$

Now I have only
variable (y) and
can solve for (y

$2 - 6y + 2y = 6$

$2 - 4y = 6$

$-2 \qquad -2$

$\frac{-4y}{-4} = \frac{4}{-4}$

$y = -1$

(x, y)

a. (-2, 1)

b. (3, 1)

c. (4, -1)

d. (7, -2)

2. Solve the system of linear equations:

$$\begin{cases} x + 3y = 1 \\ 2x + 2y = 6 \end{cases}$$

x =

y =

3. Solve the system of linear equations:

$$\begin{cases} 0.5x - 2y = 6 \\ 3x + 8y = 16 \end{cases}$$

x =

y =

4. Marta's age is 4 less than 2 times Gavin's age. The sum of their ages is 20. The system of equations below represents Marta's age *m* and Gavin's age *g*. How old are Marta and Gavin?

$$\begin{cases} 4x - 3y = -1 \\ -2x + 5y = 11 \end{cases}$$

$x =$

$y =$