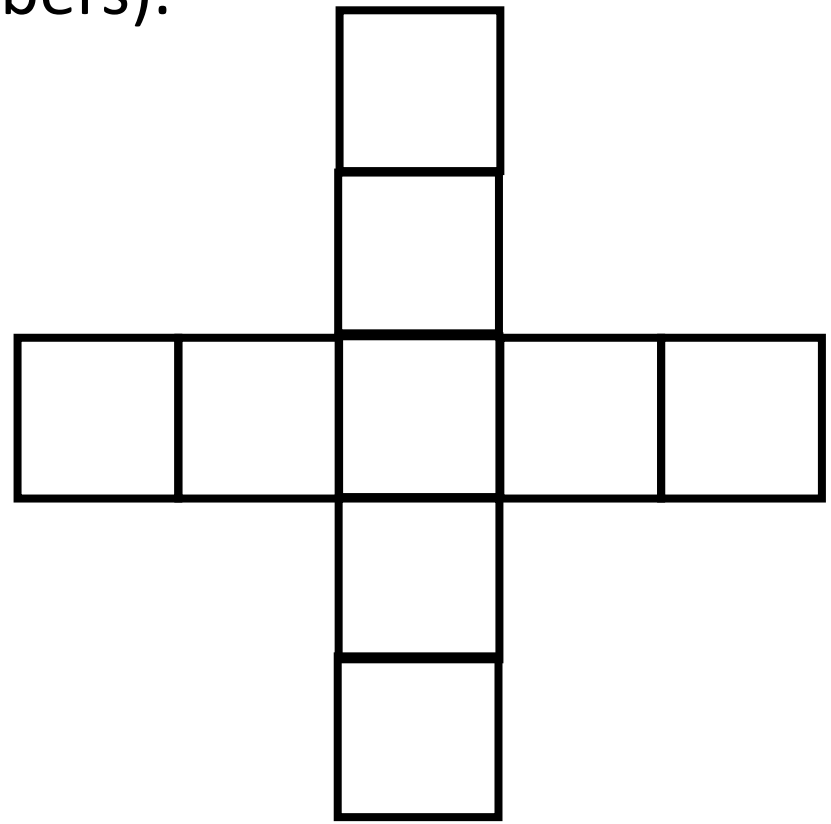
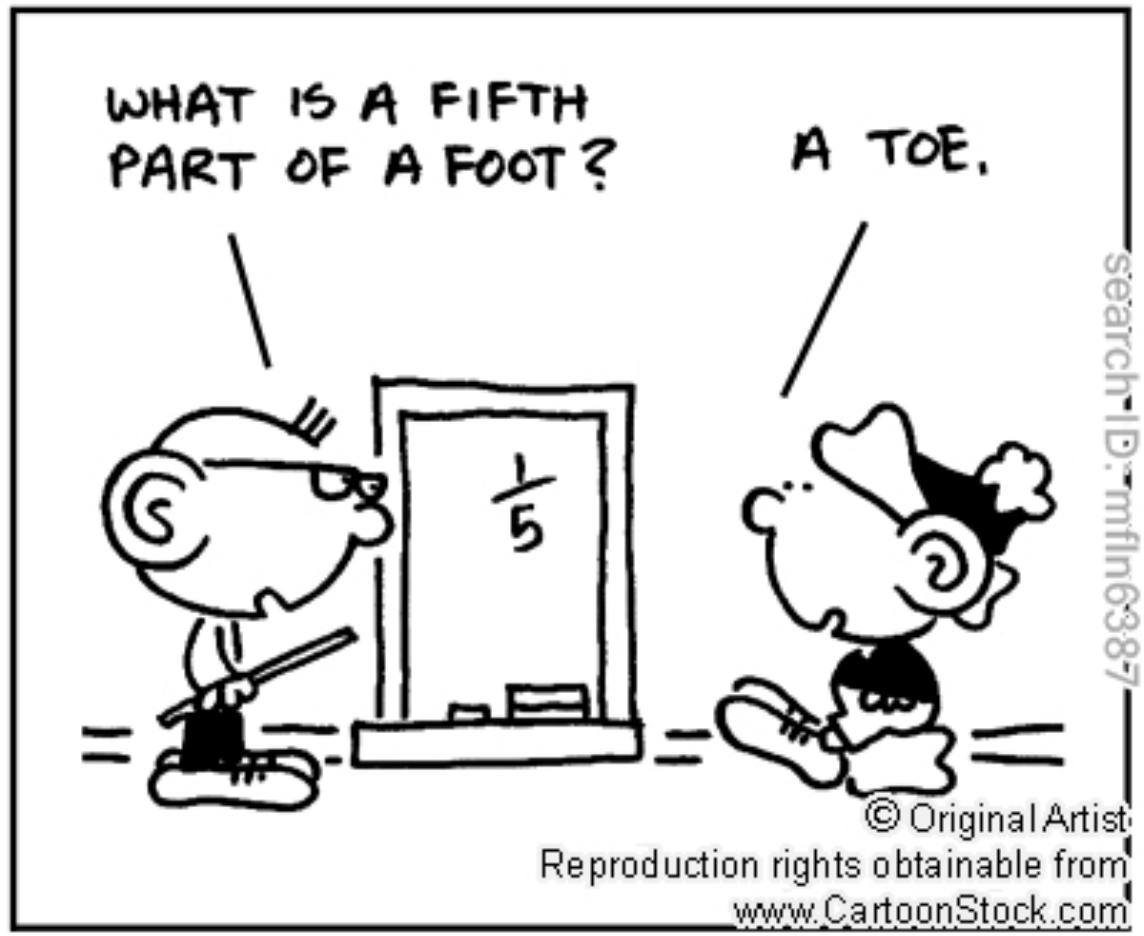
Day 3: Fractions, Decimals, Exponents & Squares/Cubes

Warm-up

How many boxes are there? Make an equation using integers (whole numbers).



Fractions!


WHAT IS A FIFTH PART OF A FOOT?
A TOE.
1/5
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Fractions: the basics

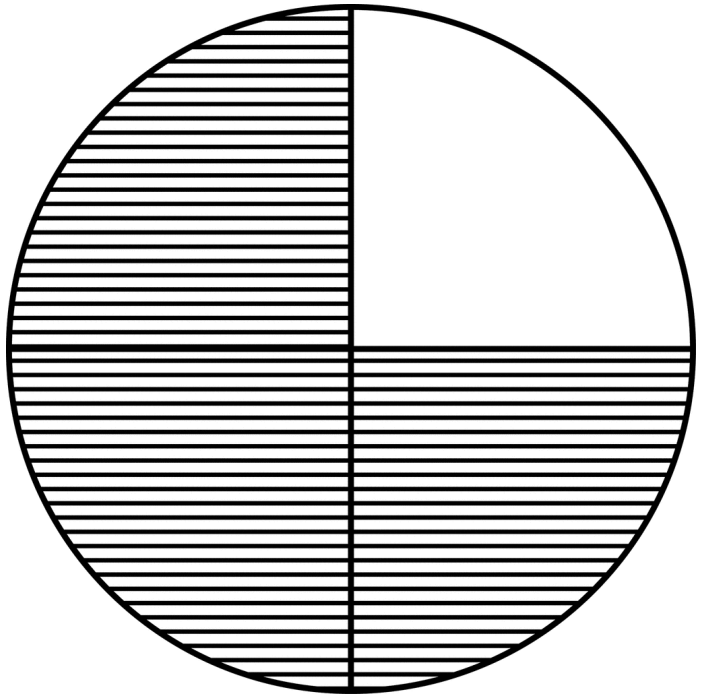
Fractions are made of two numbers:

$$\frac{\textit{Numerator}}{\textit{Denominator}}$$

We say the numerator first: $\frac{3}{4}$ is pronounced “three fourths.”

Fractions: the basics

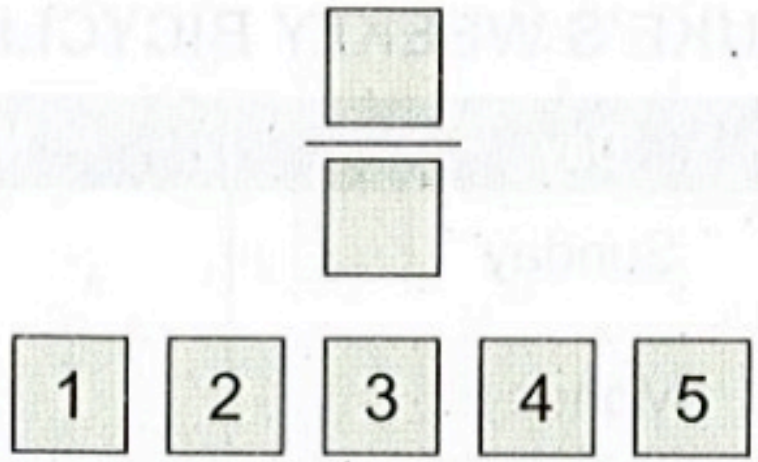
The fraction $\frac{3}{4}$ means that the whole is split into four pieces, and three of them are present.



How should we write the following fractions?

- Two tenths
- One half
- Five eighths
- Eight thirds

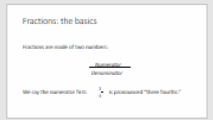
2. Two out of every five students in a high school are male. What fraction of the high school students are male?

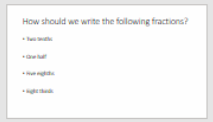


3. There are 64 students in the school band. There are 16 trumpet players. What fraction of the band are trumpet players?

$$\frac{\square}{\square}$$

1	3	4	5	7	8





Total number

3. There are 64 students in the school band. There are 16 trumpet players. What fraction of the band are trumpet players?

$$\frac{16}{64}$$

$$\frac{\square}{\square}$$

1 3 4 5 7 8

Factors of 16: 1, 2, 4, 8, 16

$$\frac{16}{64} = \frac{16 / 4}{64 / 4} = \frac{4}{16} = \frac{4 / 4}{16 / 4} = \frac{1}{4}$$

"Reducing a fraction" means to use the smallest numbers possible (while still using whole numbers)

Reducing Fractions

1. Find the Greatest Common Factor (GCF)
2. Divide the numerator and denominator by the GCF

1. $\frac{40}{60} = \_\_\_\_$
2. $\frac{18}{27} = \_\_\_\_$
3. $\frac{15}{25} = \_\_\_\_$
4. $\frac{45}{99} = \_\_\_\_$
5. $\frac{6}{66} = \_\_\_\_$
6. $\frac{30}{120} = \_\_\_\_$

40 / 10 = 4

60 / 10 = 6

4/6

Reducing Fractions

6 / 6 = 1

66 / 6 = 11

1. Find the Greatest Common Factor (GCF)
2. Divide the numerator and denominator by the GCF

1 $\frac{40}{60} = \frac{2}{3}$

2 $\frac{18}{27} = \frac{2}{3}$

3 $\frac{15}{25} = \frac{3}{5}$

4 $\frac{45}{99} = \frac{5}{11}$

5 $\frac{6}{66} = \frac{1}{11}$

6 $\frac{30}{120} = \frac{/30}{/30} \frac{1}{4}$

$\frac{30}{120} \frac{/10}{=} \frac{3}{12} \frac{/3}{=} \frac{1}{4}$

2, 3, 10, 15, 30

Reducing fractions, in a word problem

5. Anna withdrew $50 from her checking account. She spent $28 on a pair of shoes. What fraction of her money does Anna have left?

$$\frac{\square}{\square}$$

11	14	25	28	50

Finding a Common Denominator

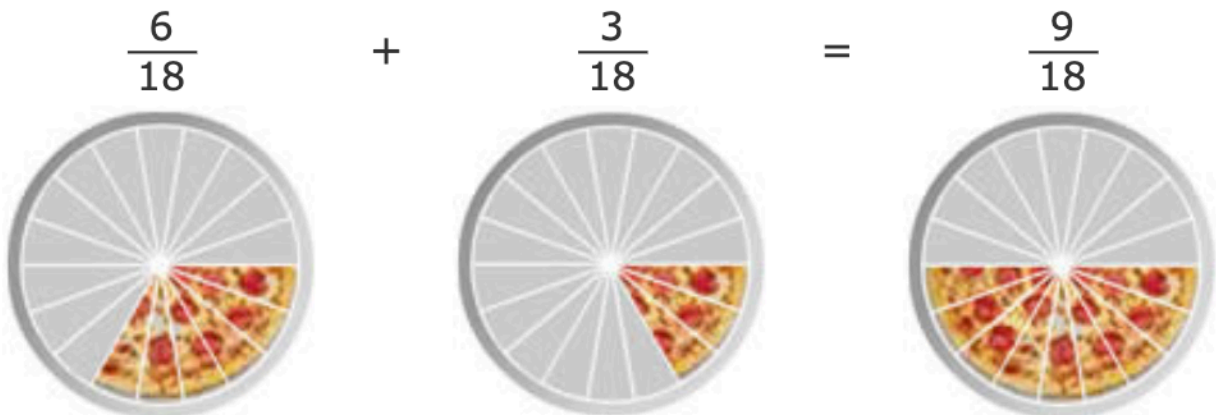
But what should the new denominator be?

One simple answer is to multiply the current denominators together:

$$3 \times 6 = 18$$

So instead of having 3 or 6 slices, we will make **both** of them have **18 slices**.

The pizzas now look like this:



They now have common denominators (but not the *least* common denominator)

(Read more about Common Denominators.)

Least Common Denominator

That is all fine, but 18 is a lot of slices ... can we do it with **fewer slices**?

Here is how to find out:

$\frac{1}{3}$ List multiples of 3: 3, 6, 9, 12, 15, 18, 21, .

$\frac{1}{6}$ List multiples of 6: 6, 12, 18, 24, ...

Now find the **smallest number** that is the same:

multiples of 3: 3, **6**, 9, 12, 15, 18, 21, ...

multiples of 6: **6**, 12, 18, 24, ...

The answer is 6, and that is the **Least** Common Denominator.

What is a …
… Denominator?

The **denominator** is the bottom number in a fraction.

It shows how many equal parts the item is divided into

$$\frac{3}{4}$$ ← Numerator

$$\frac{3}{4}$$ ← Denominator

… Common Denominator?

When the denominators of two or more fractions are the **same**, they have **Common Denominators**.

numerators

$$\frac{2}{5} + \frac{1}{5}$$

denominators

These denominators are common (the same)

... Least Common Denominator?

it is the **smallest** of all the common denominators.

Why?

Why do we want common denominators?

Because we **can't** add fractions with different denominators:

$\frac{1}{3} + \frac{1}{6} = ?$

Before we can add them we must make the **denominators the same**.

So let us try using it!

We want both fractions to have 6 slices:

- When we multiply top and bottom of $\frac{1}{3}$ by 2 we get $\frac{2}{6}$
- $\frac{1}{6}$ already has a denominator of 6

And our question now looks like:

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

One last step is to simplify the fraction (if possible). In this case 3/6 is simpler as 1/2:

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Adding and Subtracting Fractions

$$\frac{1}{3} + \frac{1}{4}$$

 + = ?

Adding and Subtracting Fractions

$$\frac{1}{3} + \frac{1}{4}$$

$= ?$

Step one: is there a common denominator?

$3 * 4 = 12$

$$\frac{1}{3} \; {}^{*4} = \frac{4}{12}$$

$$\frac{1}{4} \; {}^{*3} = \frac{3}{12}$$

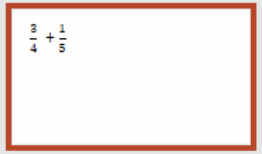
$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Step two: when the denominators are the same, add or subtract the numerators

$$\frac{3}{4} + \frac{1}{5}$$





$$\frac{3}{4} + \frac{1}{5} \quad \begin{matrix} *4 \\ *4 \end{matrix}$$

$$\frac{15}{20} + \frac{4}{20} = \frac{19}{20}$$

Step one: find a common denominator

4 * 5 is 20

Step one, part two: change the fractions to use the common denominator (20)

Step two: when the denominators are the same, THEN add or subtract the numerators

Step two, part two: can I simplify (reduce) my answer?

VARIOUS STUDENTS' HOMEWORK COMPLETION

Student	Fraction Of Homework Completed
Dara	$\frac{2}{5}$
Natalia	$\frac{7}{10}$
Miguel	$\frac{1}{2}$
Ethan	$\frac{9}{10}$
Walt	$\frac{4}{5}$

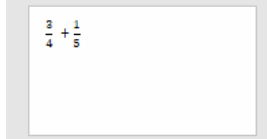
6. List the students from the table above in order of the amount of homework they completed, beginning with the one that completed the most.

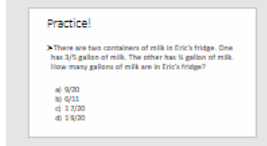
1 [ ] 2 [ ] 3 [ ]

4 [ ] 5 [ ]

7. The fraction of homework completed by which two students in the table equals the fraction of homework completed by Ethan?

[ ] and [ ]



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6. List the students from the table above in order of the amount of homework they completed, beginning with the one that completed the most.

Ethan, Walt, Natalia, Miguel, Dara

7. The fraction of homework completed by which two students in the table equals the fraction of homework completed by Ethan?

_____ and _____

What is common denominator between 2, 5, and 10?

5 Dara: $\frac{2 * 2}{2 * 5} = \frac{4}{10}$

3 Natalia $\frac{1 * 7}{1 * 10} = \frac{7}{10}$

4 Miguel $\frac{5 * 1}{5 * 2} = \frac{5}{10}$

1 Ethan $\frac{1 * 9}{1 * 10} = \frac{9}{10}$

2 Walt $\frac{2 * 4}{2 * 5} = \frac{8}{10}$

Practice!

- There are two containers of milk in Eric's fridge. One has 3/5 gallon of milk. The other has ¾ gallon of milk. How many gallons of milk are in Eric's fridge?

a) 9/20
b) 6/11
c) 1 7/20
d) 1 9/20

Multiplying and Dividing Fractions

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$$

take the reciprocal of the divisor

$$\frac{4}{7} \div 2 = \frac{4}{7} \times \frac{1}{2} = \frac{4 \times 1}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$$

Mixed Fractions

*(Also called "**Mixed Numbers**")*

$1\frac{3}{4}$

(one and three-quarters)

A Mixed Fraction is a whole number and a proper fraction combined.

Such as $1\frac{3}{4}$

Examples

$2\frac{3}{8}$	$7\frac{1}{4}$	$1\frac{14}{15}$	$21\frac{4}{5}$

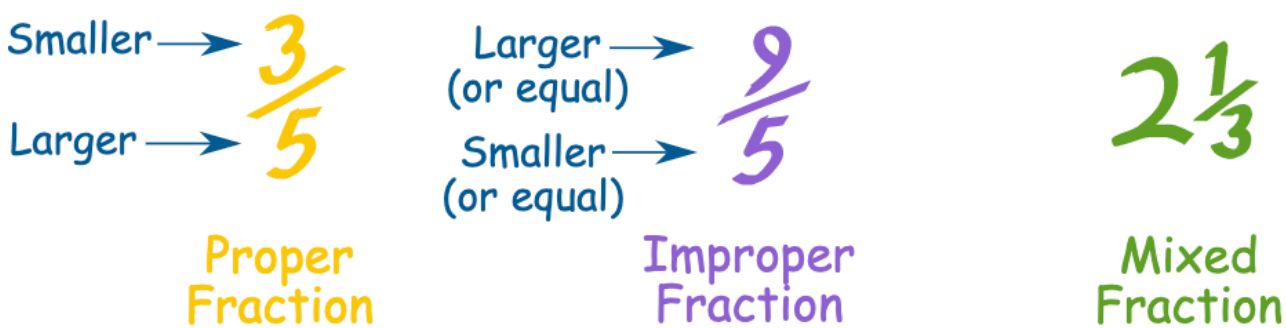
See how each example is made up of a whole number **and** a proper fraction together? That is why it is called a "mixed" fraction (or mixed number).

Names

We can give names to every part of a mixed fraction:

Three Types of Fractions

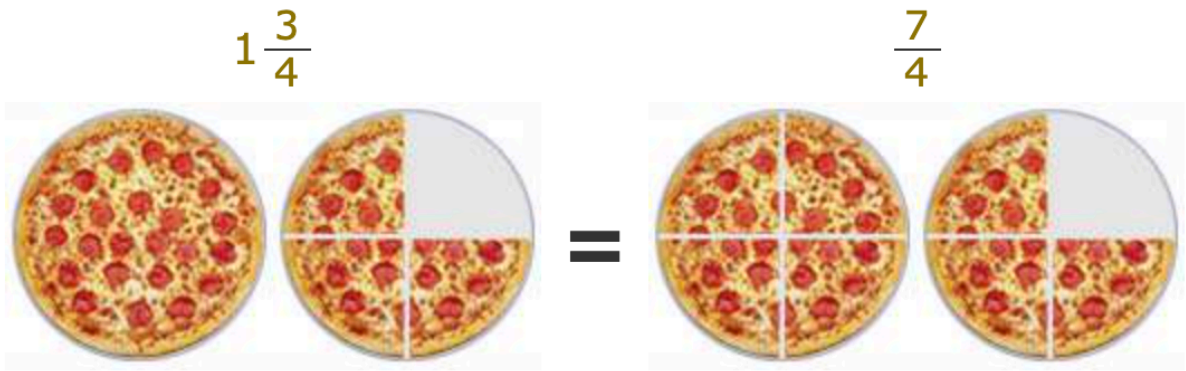
There are three types of fraction:



Mixed Fractions or Improper Fractions

We can use either an improper fraction or a mixed fraction to show the same amount.

For example $1\frac{3}{4} = \frac{7}{4}$, as shown here:



Converting Improper Fractions to Mixed Fractions

To convert an improper fraction to a mixed fraction, follow these steps:

- Divide the numerator by the denominator.
- Write down the whole number answer
- Then write down any remainder above the denominator.

Example: Convert $\frac{11}{4}$ to a mixed fraction.

Divide:

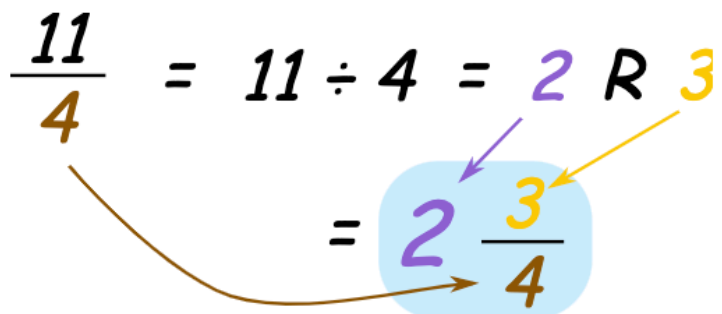
$11 \div 4 = 2$ with a remainder of 3

Write down the 2 and then write down the remainder (3) above the denominator (4).

Answer:

$$2\ \frac{3}{4}$$

That example can be written like this:



Converting Mixed Fractions to Improper Fractions

To convert a mixed fraction to an improper fraction, follow these steps:

- Multiply the whole number part by the fraction's denominator.
- Add that to the numerator
- Then write the result on top of the denominator.

Example: Convert $3\frac{2}{5}$ to an improper fraction.

Multiply the whole number part by the denominator:

$3 \times 5 = 15$

Add that to the numerator:

$15 + 2 = 17$

Then write that result above the denominator:

$$\frac{17}{5}$$

Are Improper Fractions Bad ?

NO, they aren't bad!

For mathematics they are actually **better** than mixed fractions. Because mixed fractions can be confusing when we write them in a formula: ***should the two parts be added or multiplied?***

Mixed Fraction: What is: $1 + 2\frac{1}{4}$?

Is it: $1 + 2 + \frac{1}{4} = 3\frac{1}{4}$ **?**

Or is it: $1 + 2 \times \frac{1}{4} = 1\frac{1}{2}$ **?**

Improper Fraction: What is: $1 + \frac{9}{4}$?

It is: $\frac{4}{4} + \frac{9}{4} = \frac{13}{4}$ ✓

But, for **everyday use**, people understand mixed fractions better.

Example: It is easier to say "I ate $2\frac{1}{4}$ sausages", than "I ate $\frac{9}{4}$ sausages"

Ratios and Proportions, using fractions

DIRECTIONS: Read each question. Then use the drag-and-drop options to complete each proportion and answer.

13. For each $5 given to a charity by an individual, the Bay Company will give $15 to that same charity. If individual contributions total $275, how many dollars will the Bay Company contribute?

$$\frac{\$5}{\$\square} = \frac{\$\square}{\$\square} = \$\square$$

x	15	91	125	275	825

Ratios and Proportions, using fractions

14. In a school, the ratio of students to teachers is 14 to 1. If there are 406 students, how many teachers work at the school?

$$\frac{14}{\square} = \frac{\square}{\square} = \square$$

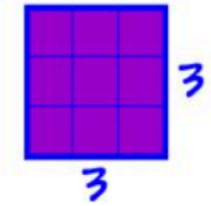
x	1	28	29	203	406

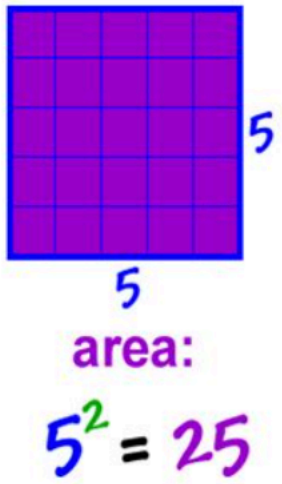
Warm-up

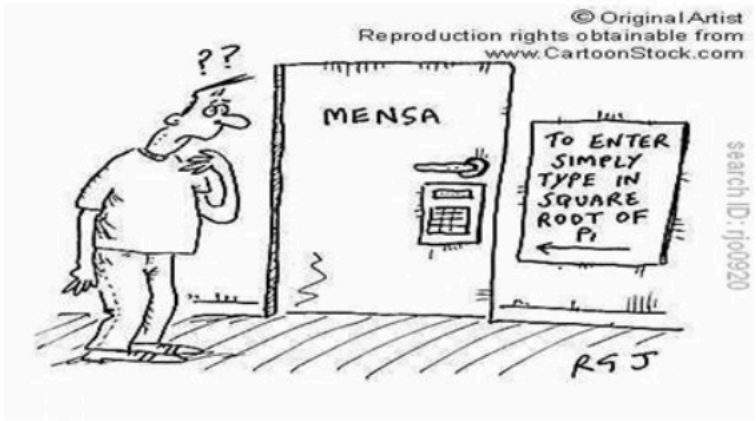
- *Approx. distance from the Earth to the moon: 2.389 x 10^5 mi (3.844 x 10^5 km) =*
- *Approx. distance from the Earth to the sun: 9.3x10^7mi (1.496x10^8km) =*

Exponents, Square Roots, Cubes

- When a number or variable is multiplied by itself, the result is called the square of that number or variable.
- Squaring the number 5, for example, is finding the product of 5 x 5= 25, this product is written as 5^2, where the ^{2} indicated that the product is composed of two factors of 5.

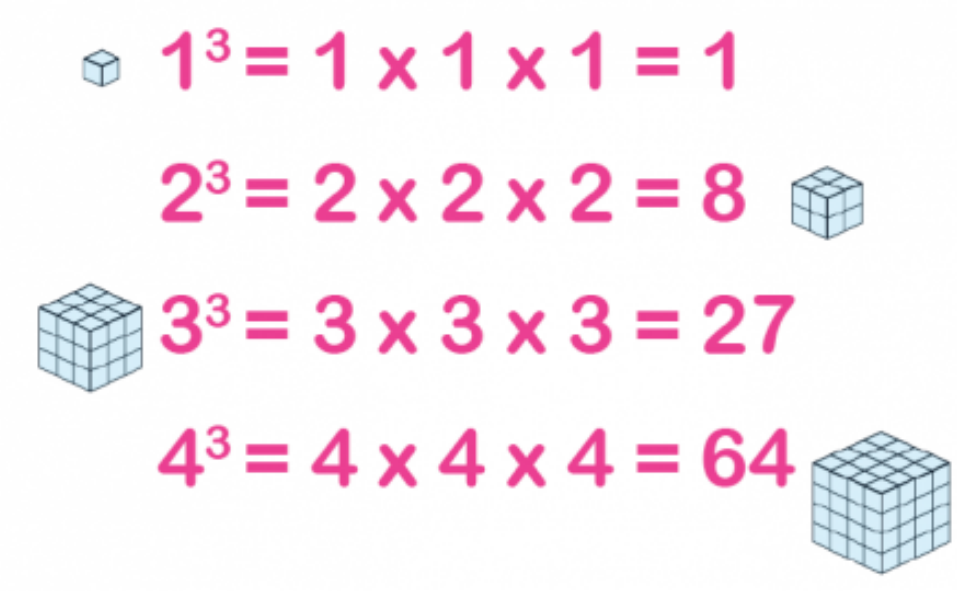






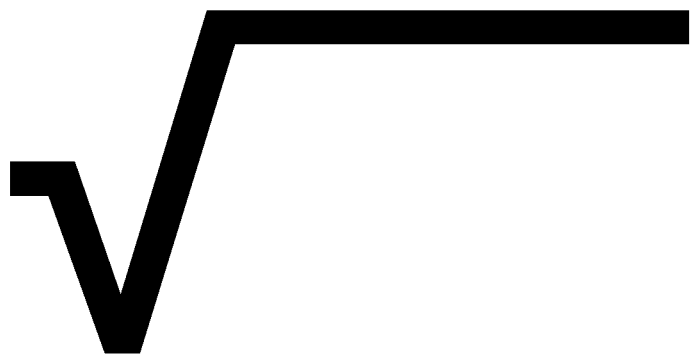
Exponents, Square Roots, Cubes

- When a number or variable is multiplied by itself an additional time, the result is called the cube of the number or variable. For example, the cube of 5 is 5 x 5 x 5 = 125; the product is written as 5^3



Exponents, Square Roots, Cubes

- To find the square root of a number, find the number that, when squared, equals the given number. The cube root of a number is that number which, when cubed, equals the given number. Square and cube roots are indicated by radical signs:



Exponents, Square Roots, Cubes

- The square roots of a negative number is positive; if two numbers differ only in their sign, their squares are both positive and equal. The square root of a positive number can, as a result, have two values. Since there are no real numbers that, when multiplied by themselves, give a negative number, square roots of negative numbers are undefined when dealing with real numbers.
- The cube of a negative number is negative. As a result, the cube root of a negative number exists, is negative, and is equal in magnitude to the cube root of the absolute value of the number.

$(-3)^2$ and -3^2

Simplify $(-3)^2$ and -3^2

$= (-3)(-3)$ $= -3 \cdot 3$

$= 9$ ✓ $= -9$ ✓

Practice!

- The length of a square can be determined by finding the square root of its area. If a square has an area of 81 m^2, what is the length of the square?

a) 8 m
b) 8.5 m
c) 9 m
d) 9.5 m

Practice!

- Mark multiplied a number by itself. He found a product of 30. What is the number, rounded to the nearest tenth?

a) 4.5
b) 5.4
c) 5.5
d) 15

Practice!

- A gallon has a volume of 231 cubic inches. If a gallon of milk was sold in a perfectly cubical container, to the nearest tenth of an inch how high would the container be?

a) 6 inches
b) 6.1 inches
c) 6.2 inches
d) 6.3 inches