Day 10:
Quadratic Graphs and Functions

Learning Objectives

1. Identify and define quadratic functions and equations

2. Identify and use properties of quadratic functions and their graphs

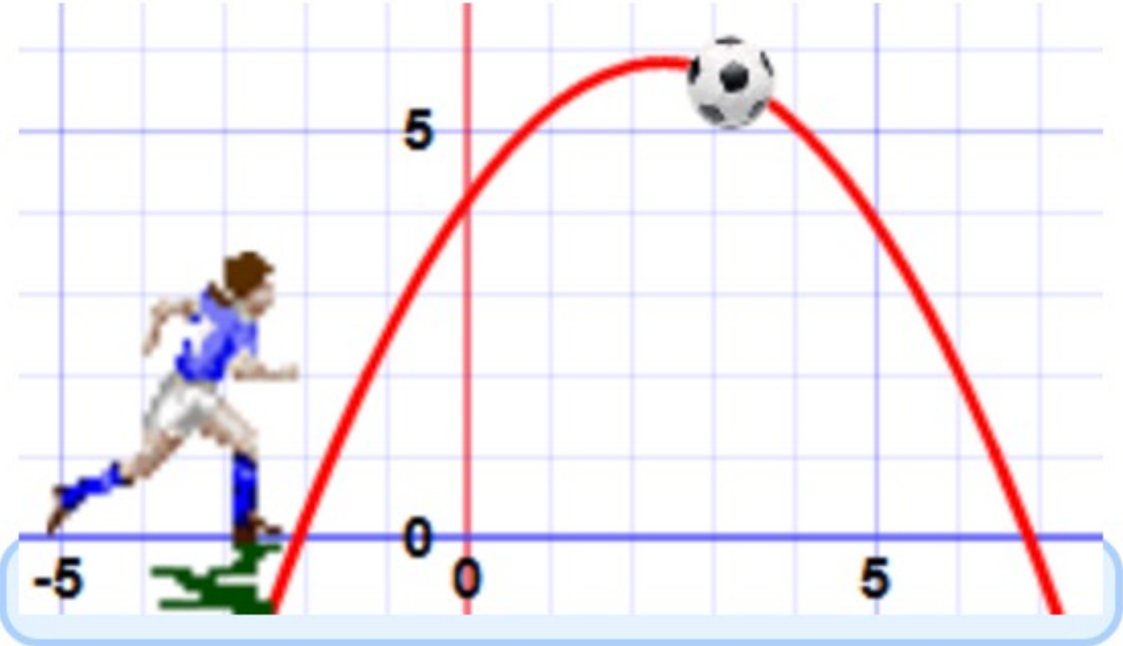
3. Solve applications of quadratic functions

An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Quadratic Equations make nice curves, like this one:



An example of a **Quadratic Equation**:

n exponent of 2 makes it a
uadratic equation
remember that in a linear
quation we didn't have
xponents)

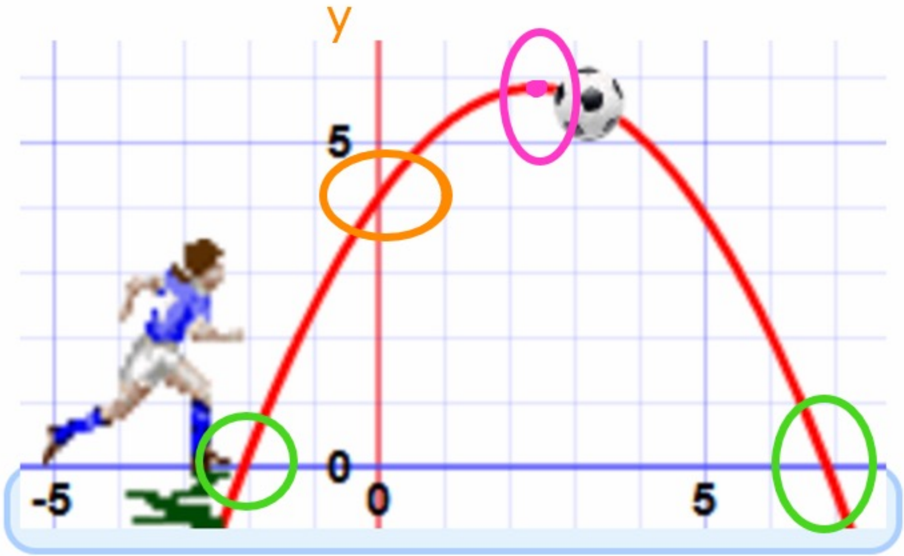
this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Quadratic Equations make nice curves, like this one:

What are the x and y intercepts of the graph?

What is the vertex of the graph?

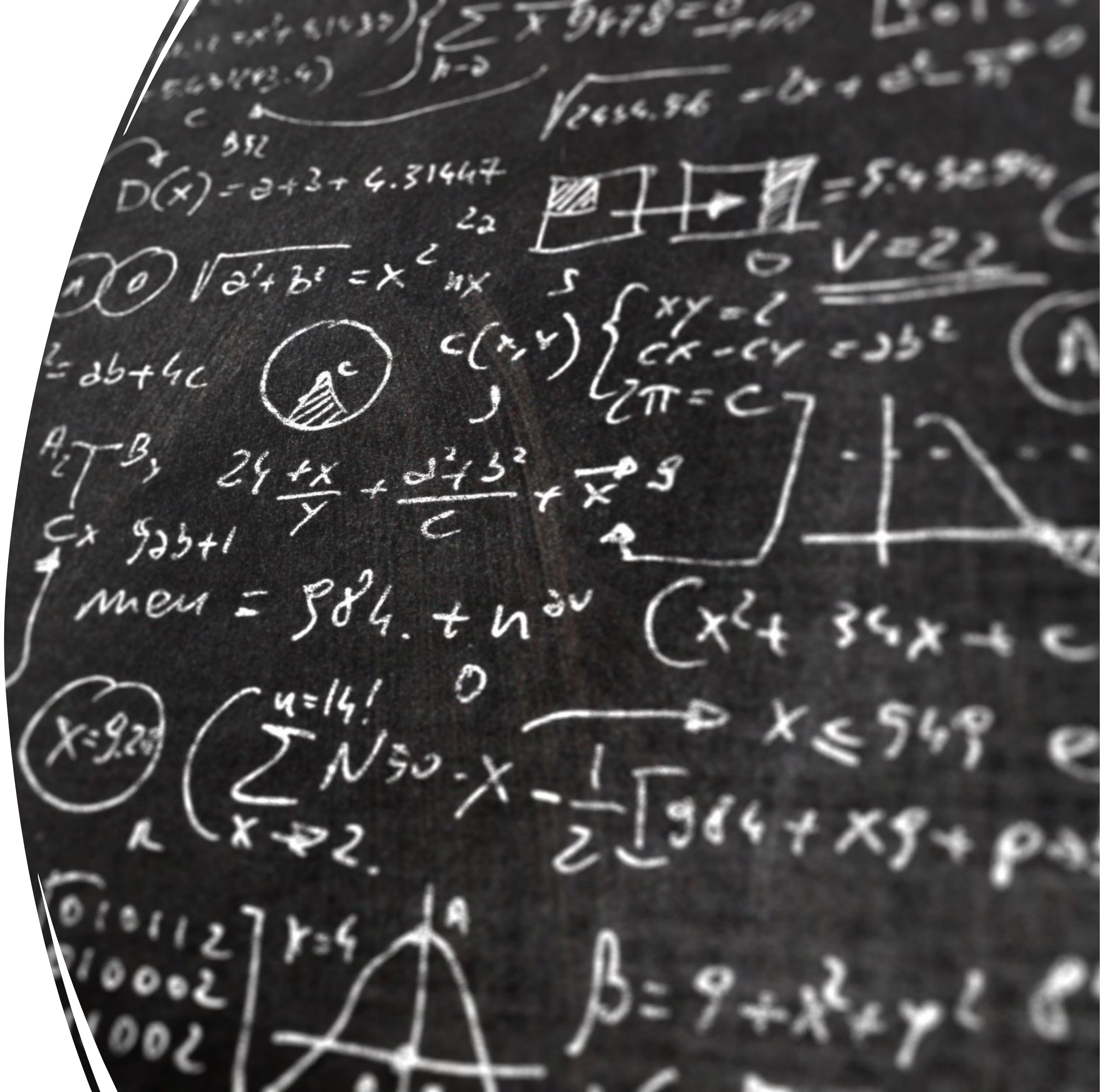


ROOTS (x intercepts) -- there might be two of these

Examples

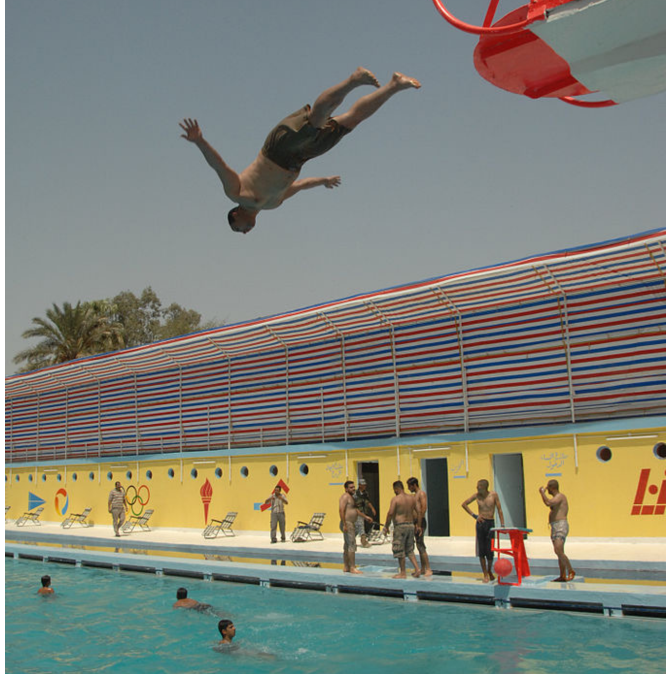
$2x^2 + 5x + 3 = 0$		In this one **a=2**, **b=5** and **c=3**
$x^2 - 3x = 0$		This one is a little more tricky: • Where is **a**? Well **a=1**, as we don't usually write "$1x^2$" • **b = −3** • And where is **c**? Well **c=0**, so is not shown.
$5x - 3 = 0$		**Oops!** This one is **not** a quadratic equation: it is missing x^2 (in other words **a=0**, which means it can't be quadratic)

Graphing Quadratic Equations



Applications of Quadratic Functions

- Throwing a ball, shooting a cannon, diving from a platform and hitting a golf ball are all real-world examples of situations that can be modeled by quadratic functions.



Standard Form

The **Standard Form** of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

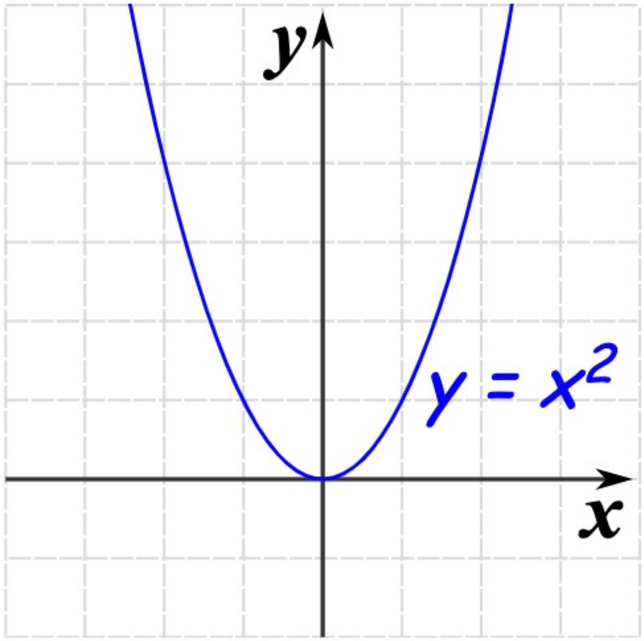
- **a**, **b** and **c** are known values. **a** can't be 0.
- "**x**" is the **variable** or unknown (we don't know it yet).

The Simplest Quadratic

The simplest Quadratic Equation is:

$$f(x) = x^2$$

And its graph is simple too:



This is the curve $f(x) = x^2$

It is a parabola.

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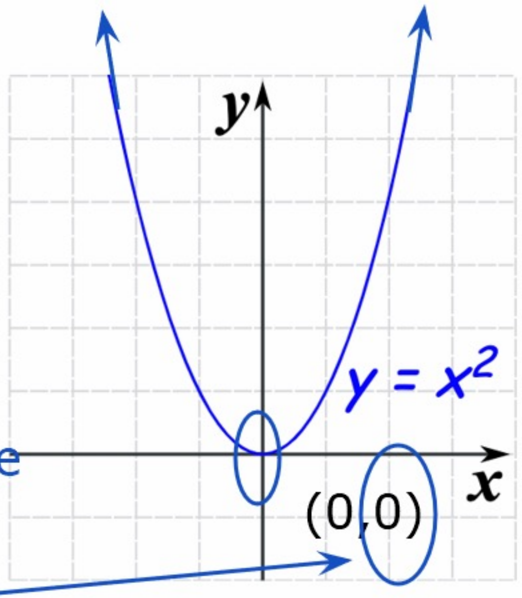


y-value

Does this graph have a maximum or a minimum?

Minimum, because it open upward

What is the minimum value of this equation?

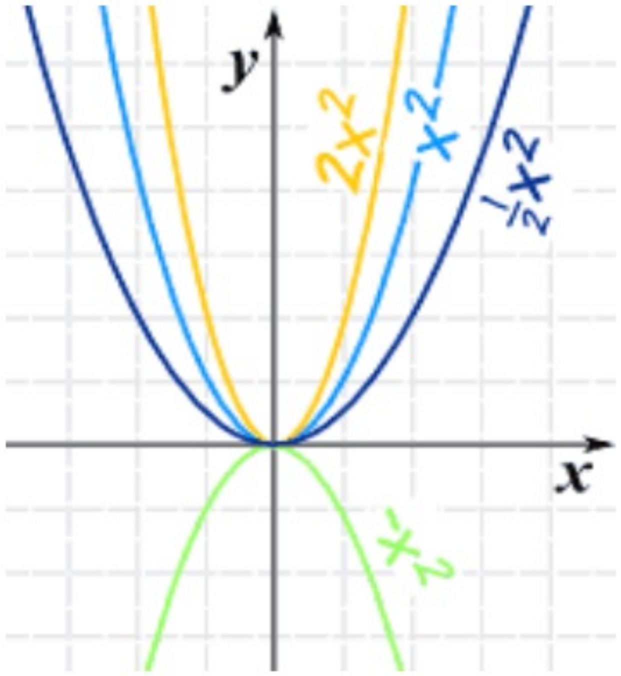
The minimum value is zero.

This is the curve $f(x) = x^2$

It is a parabola.

Now let us see what happens when we introduce the "a" value:

$$f(x) = ax^2$$



- Larger values of **a** squash the curve inwards
- Smaller values of **a** expand it outwards
- And negative values of **a** flip it upside down

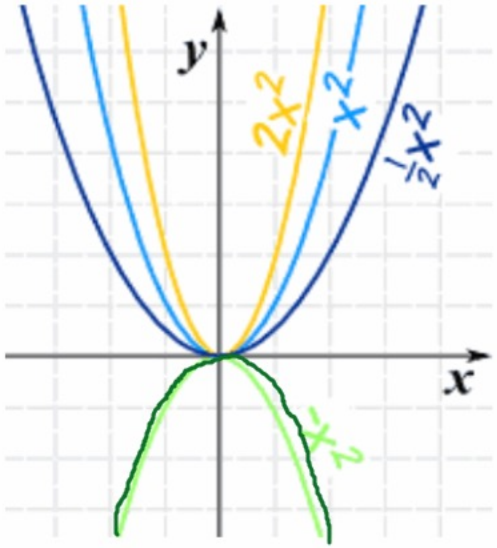
Now let us see what happens when we introduce the "a" value:

$f(x) = ax^2$

y = 2x^2 is steeper or narrower, because (a) is larger

y = (1/2)x^2 is shallower, or wider, because (a) is smaller

y = -x^2 is flipped upside down, because (a) is negative



2^2 = 4
3^2 = 9

-(2^2) = -4
-(3^2) = -9

-x^2

- Larger values of **a** squash the curve inwards
- Smaller values of **a** expand it outwards
- And negative values of **a** flip it upside down

Important Tip:

- In the standard form: $ax^2 + bx + c = 0$

- **a** changes how wide or narrow the curve is (negative a flips it upside down)
- **b** shifts it left or right
- **c** is the y-intercept

To graph a quadratic equation, you will want to know the highest or lowest point on the parabola. This point is known as the **vertex**. It shows the **maximum** or **minimum** value of the equation.

For example, consider throwing a football through the air. The path it takes through the air is a parabola. Natural questions to ask are:

- "When does the football reach its maximum height?“

    This is the x-value of the vertex

- "How high does the football get?

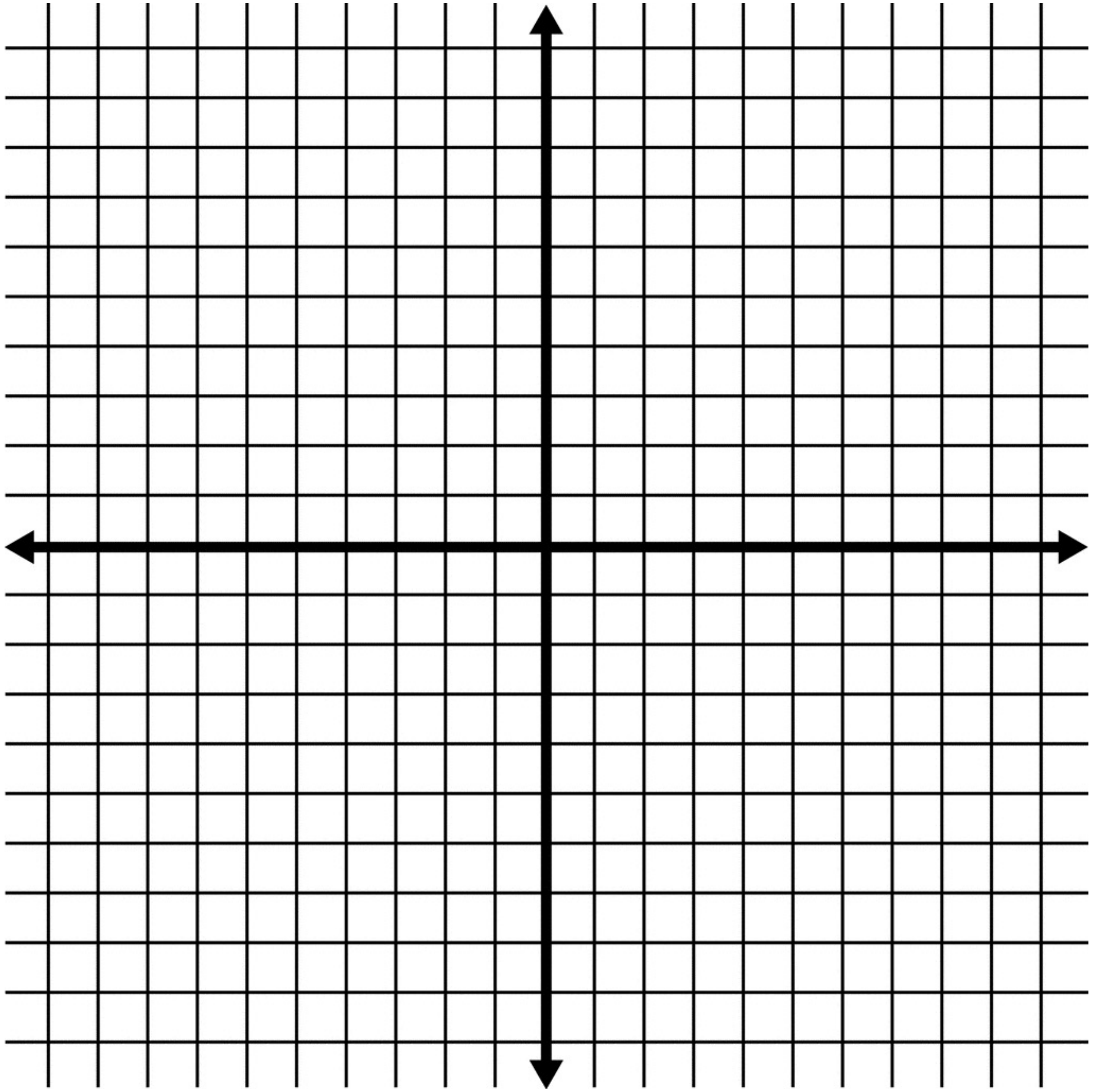
    This is the y-value of the vertex

- "When does the football reach its maximum height?“

  This is the x-value of the vertex

- "How high does the football get?

  This is the y-value of the vertex



If you know the equation for the function that models the situation, you can find the vertex. If the function is

$$\boldsymbol{y = ax^2 + bx + c}$$

the x-coordinate of the vertex will be

$$\boldsymbol{\frac{-b}{2a}}$$

The y-coordinate of the vertex can be found by substituting the x-coordinate into the function. In the case of the football:

- The x-coordinate of the vertex will give you the time when the football is at its maximum height.
- The y-coordinate will give you the maximum height.

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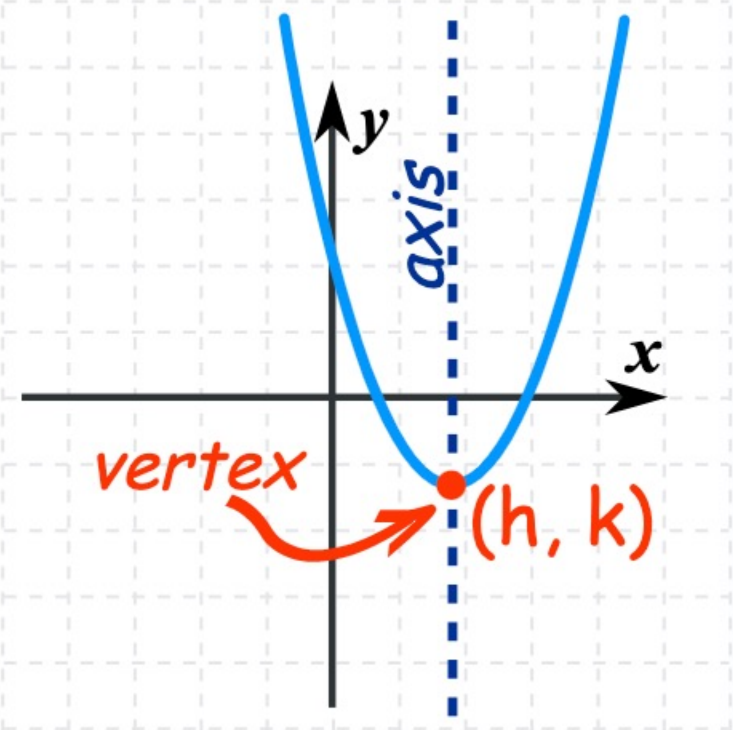
- The x-coordinate of the vertex will give you the time when the football is at its maximum height.
- The y-coordinate will give you the maximum height.

But Why?

The wonderful thing about this new form is that h and k show us the very lowest (or very highest) point, called the **vertex**:

And also the curve is symmetrical (mirror image) about the **axis** that passes through **x=h**, making it easy to graph



So ...

- **h** shows us how far left (or right) the curve has been shifted from x=0
- **k** shows us how far up (or down) the curve has been shifted from y=0

Example: Plot $f(x) = 2x^2 - 12x + 16$

First, let's note down:

- **a = 2,**
- **b = −12,** and
- **c = 16**

Now, what do we know?

- a is positive, so it is an "upwards" graph ("U" shaped)
- a is 2, so it is a little "squashed" compared to the $\mathbf{x^2}$ graph

Example: Plot ~~f(x)~~ y $= 2x^2 - 12x + 16$

Find the x-value of the vertex:

$$\frac{-b}{2a} = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$

First, let's note down:

- $a = 2$,
- $b = -12$, and
- $c = 16$

COEFFICIENTS

x value of the vertex is 3

(3, ??)

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na

y = 2(3^2) - 12 (3) + 16

y = 2(9) - 12(3) + 16

y = 18 - 36 + 16 = -2

Now we have our vertex point: (3, -2)

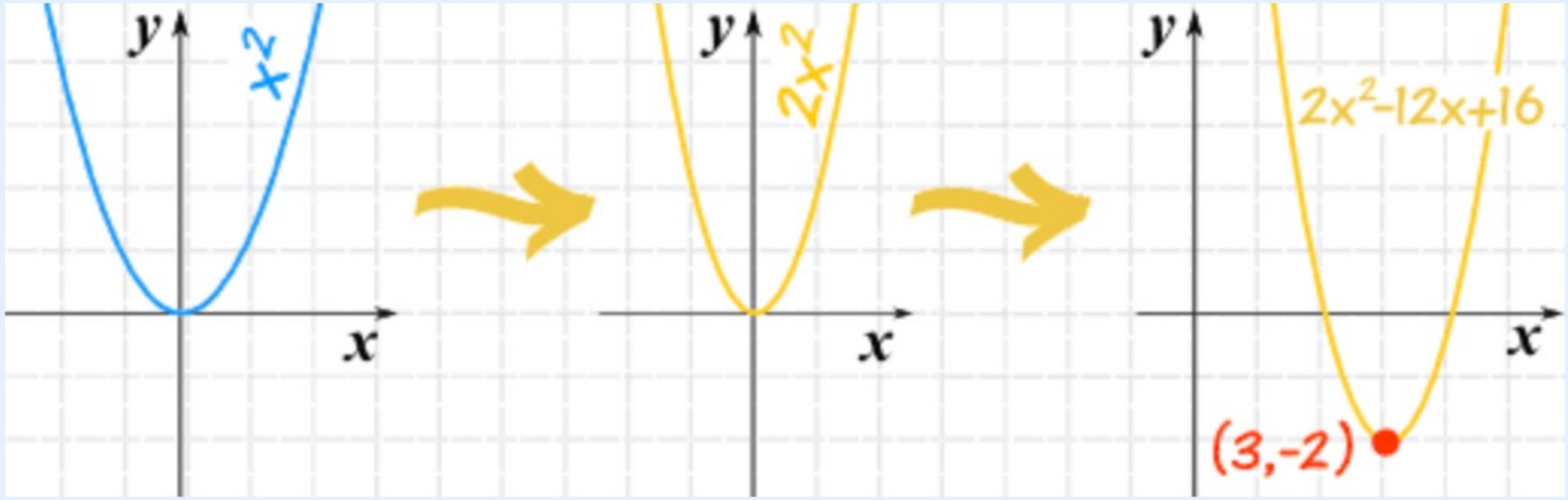
Next, let's calculate h:

$$h = -b/2a = -(-12)/(2x2) = \mathbf{3}$$

And next we can calculate k (using h=3):

$$k = \boldsymbol{f(}3\boldsymbol{)} = 2(3)^2 - 12\cdot3 + 16 = 18-36+16 = \mathbf{-2}$$

So now we can plot the graph (with real understanding!):



We also know: the **vertex** is (3,−2), and the **axis** is x=3

Next, let's calculate h:

$h = -b/2a = -(-12)/(2\times2) = \mathbf{3}$

Step one: find the x value of the vertex. x = 3

And next we can calculate k (using h=3):

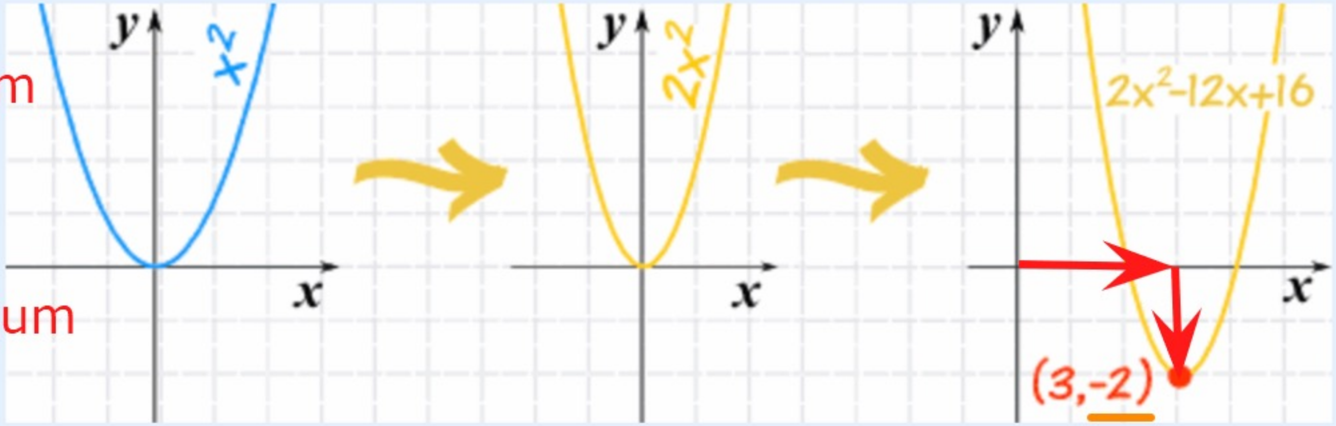
$k = \boldsymbol{f(3)} = 2(3)^2 - 12\cdot3 + 16 = 18-36+16 = \mathbf{-2}$

Step two: replace (x) with 3

So now we can plot the graph (with real understanding!):

Is there a maximum or minimum?

What is the minimum value? -2



What is the vertex; what is the minimum point? (3,-2)

We also know: the **vertex** is (3,−2), and the **axis** is x=3

The "General" Quadratic

Before graphing we **rearrange** the equation, from this:

$$f(x) = ax^2 + bx + c$$

To this:

$$f(x) = a(x-h)^2 + k$$

Where:

- $h = -b/2a$
- $k = \boldsymbol{f(}\ h\ \boldsymbol{)}$

In other words, calculate **h** (= −b/2a), then find **k** by calculating the whole equation for **x=h**

A toy rocket is fired into the air from the top of a barn. Its height (h) above the ground in yards after t seconds is given by the function

$$h(t) = -5t^2 + 10t + 20$$

- What was the initial height of the rocket?
- When did the rocket reach its maximum height?
- Sketch a graph of the function…

y = (a)(x^2) + (b)(x) + c

A toy rocket is fired into the air from the top of a barn. Its height (h) above the ground in yards after t seconds is given by the function

a = -5
b = 10
c = 20

y

$$\cancel{h(t)} = \underline{-5}t^2 + \underline{10}t + \underline{20}$$

- What was the initial height of the rocket?
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- Sketch a graph of the function...

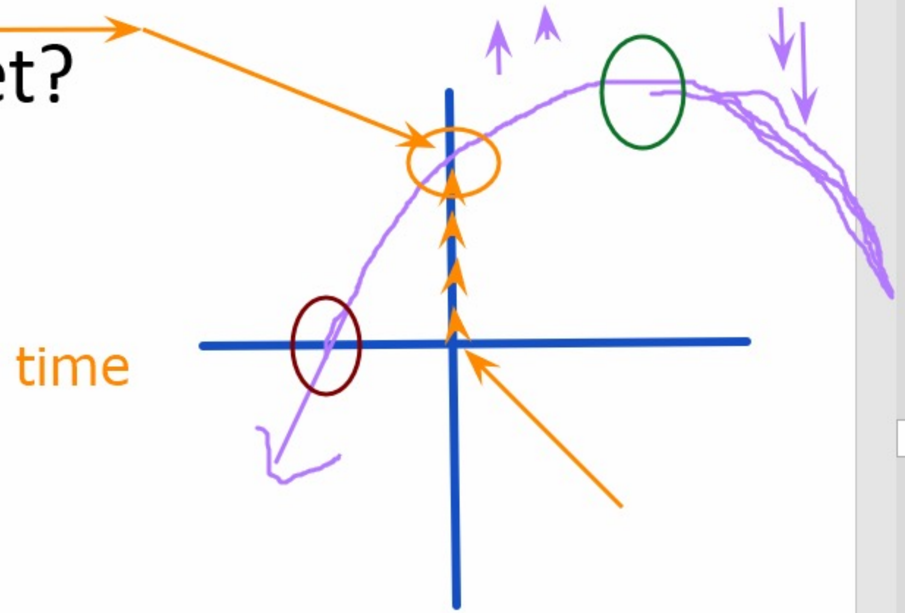
What was the initial height of the rocket?

$$h(t) = -5t^2 + 10t + 20$$

What was the initial height of the rocket?

What point on the graph am I looking for?

$$h(t) = -5t^2 + 10t + 20$$



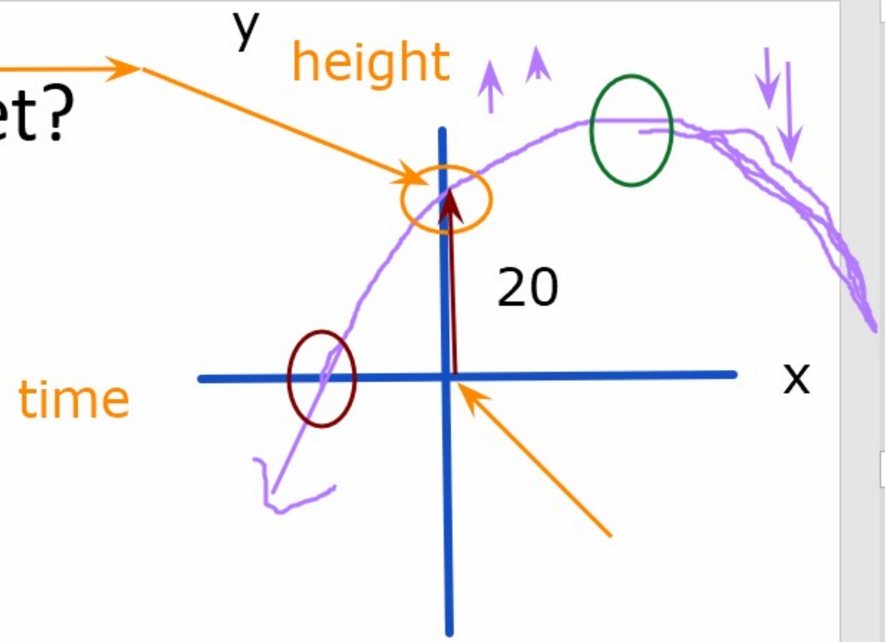
"A toy rocket is fired from a barn."
We begin at time = 0, and we want to know how high it is when time is equal to zero.

The roots tells us when the rocket hits the ground.
The y-intercept tells us the initial height (when time = 0).
The vertex tells us the maximum height.

What was the initial height of the rocket?

What point on the graph am I looking for?

$$h(t) = -5t^2 + 10t + 20$$



How do I find the height, when time is zero?

Replace (t) with zero!

$y = \cancel{-5(0^2)} + \cancel{(10)(0)} + 20 = 20$

$y = 20$

What was the initial height of the rocket?

What point on the graph am I looking for?

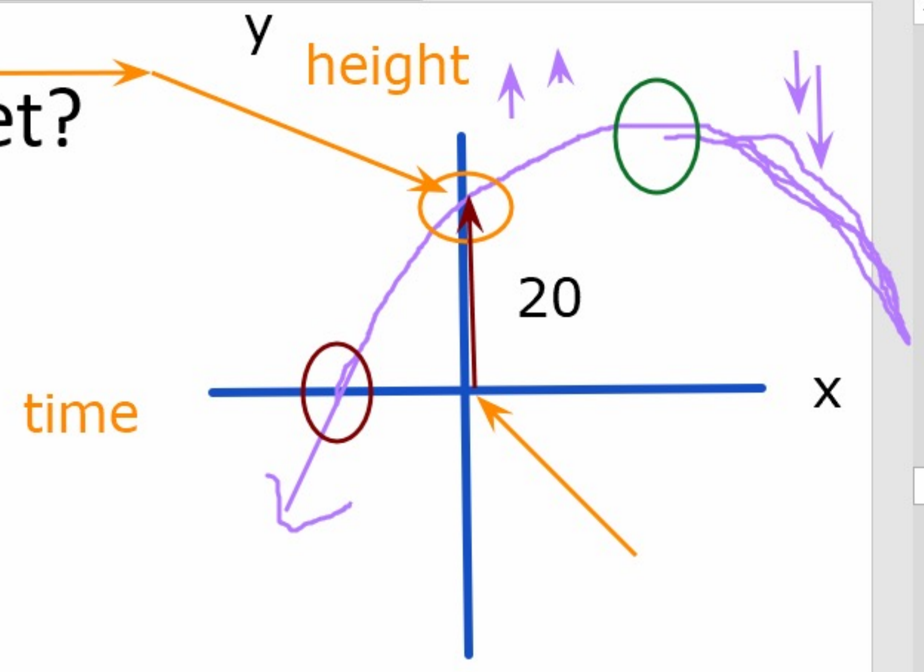
$$h(t) = -5t^2 + 10t + 20$$

How do I find the height, when time is zero?

Replace (t) with zero!

y = -5(0^2) + (10)(0) + 20 = 20

y = 20



"initial" means that time is zero (my x value is zero).

When I know my x value, I use my equation to find the y

1. The initial height of the rocket is the height from which it was fired. The time is zero.

$$h(t) = -5t^2 + 10t + 20$$
$$h(t) = -5(0)^2 + 10(0) + 20$$
$$\boxed{h(t) = 20\ yd}$$

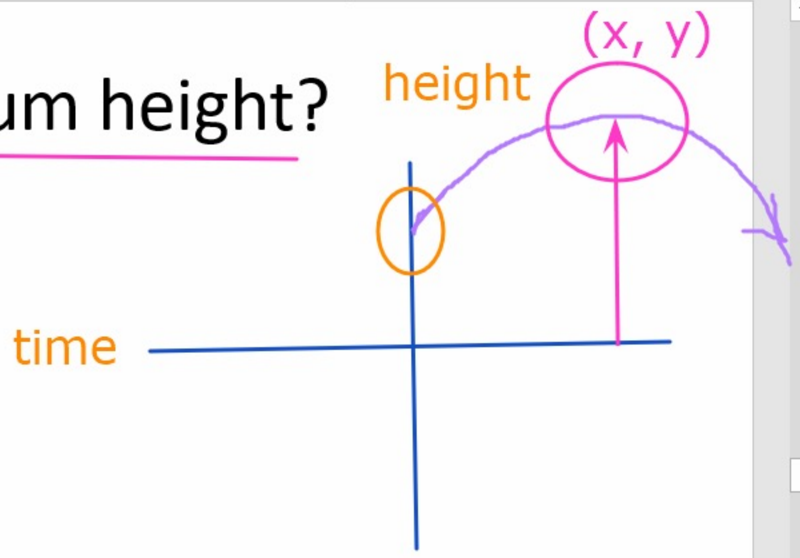
The initial height of the toy rocket is 20 yards. This is the y-intercept of the graph. The y-intercept of a quadratic function written in general form is the value of ‘c’.

When did the rocket reach its maximum height?

$$h(t) = -5t^2 + 10t + 20$$

When did the rocket reach its maximum height?

The maximum or minimum value is the y-value of the vertex.



$$h(t) = -5t^2 + 10t + 20$$

Step one: find the x coordinate of the vertex, with the formula: -b / 2a

$$\frac{-b}{2a} = \frac{-10}{2(-5)} = \frac{-10}{-10} = 1$$

(1, ?)

Step two: replace (t) with our x value, (1)
y = -5(1^2) + 10(1) + 20
y = -5 (1) + 10 (1) + 20 = -5 + 10 + 20 = 25

My vertex is a maximum.
It is located at (1, 25).

The rocket reaches its maximum height at 1 second.

2. The time at which the rocket reaches its maximum height is the x-coordinate of the vertex.

$$t = -\frac{b}{2a}$$

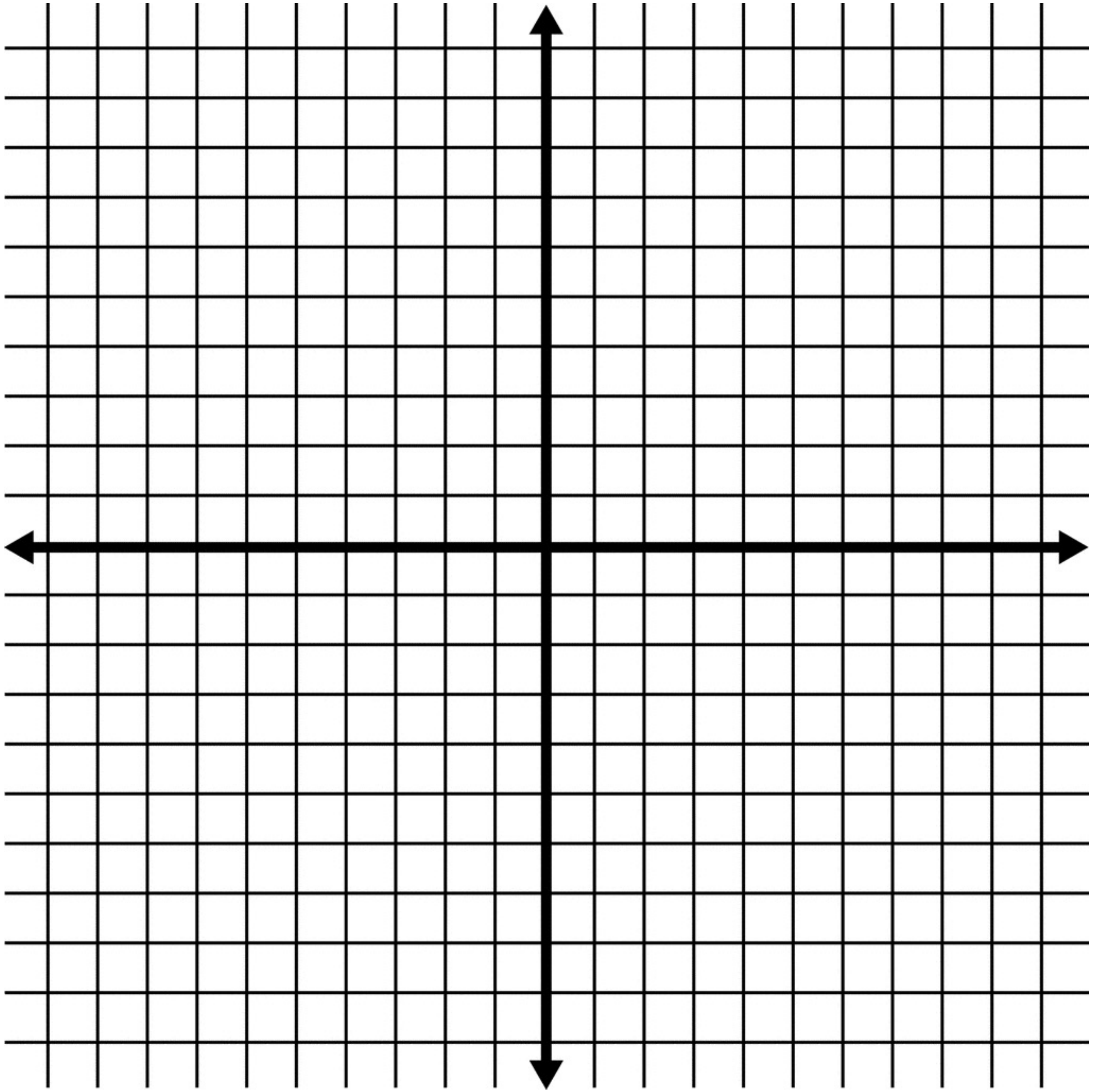
$$t = -\frac{10}{2(-5)}$$

$$\boxed{t = 1\ sec}$$

It takes the toy rocket 1 second to reach its maximum height.

Sketch a graph of the function.

$$h(t) = -5t^2 + 10t + 20$$



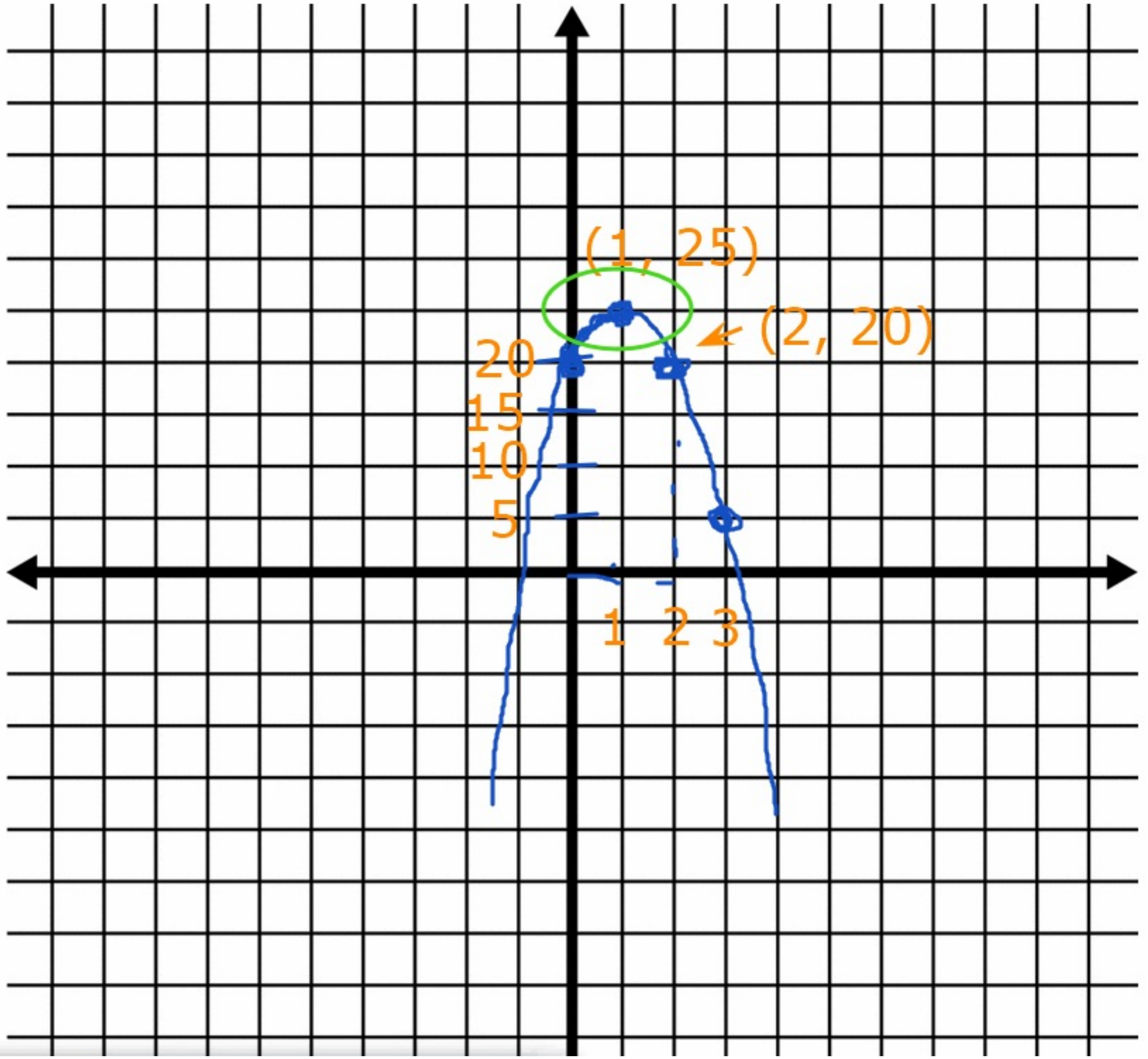
Sketch a graph of the function.

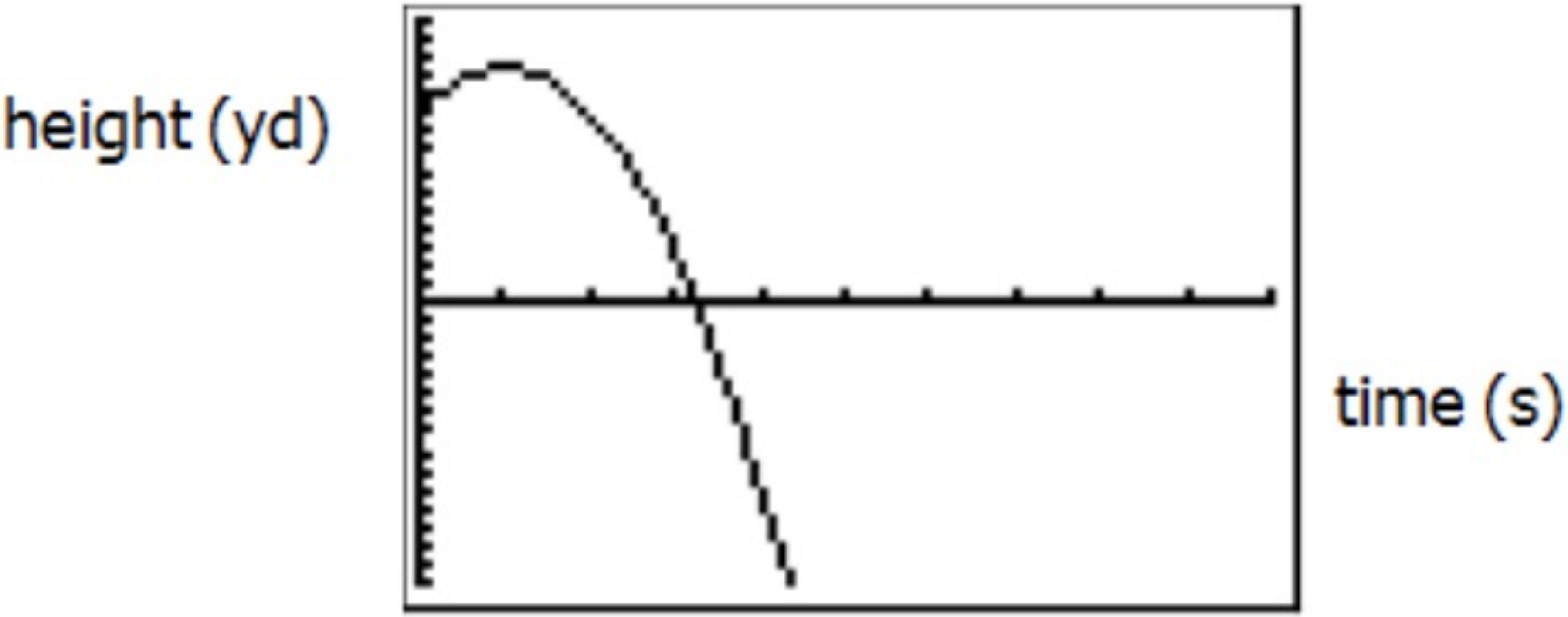
$$h(t) = -5t^2 + 10t + 20$$

The initial height is 20.
The vertex is (1, 25).

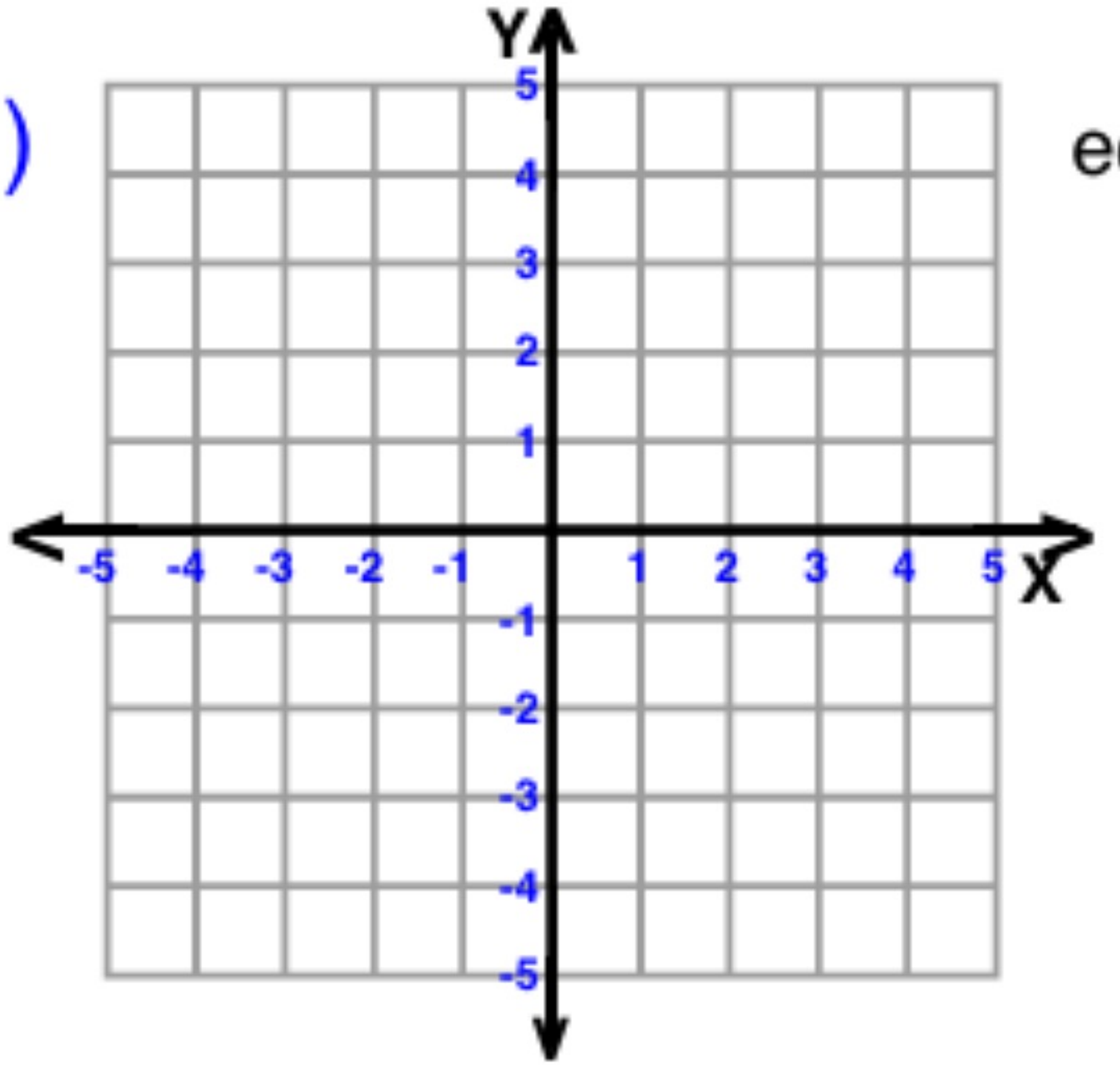
Replace (t) with 3

y = -5(3^2) + 10(3) + 20
y = -5 (9) + 10(3) + 20
y = -45 + 30 + 20
y = -45 + 50
y = 5



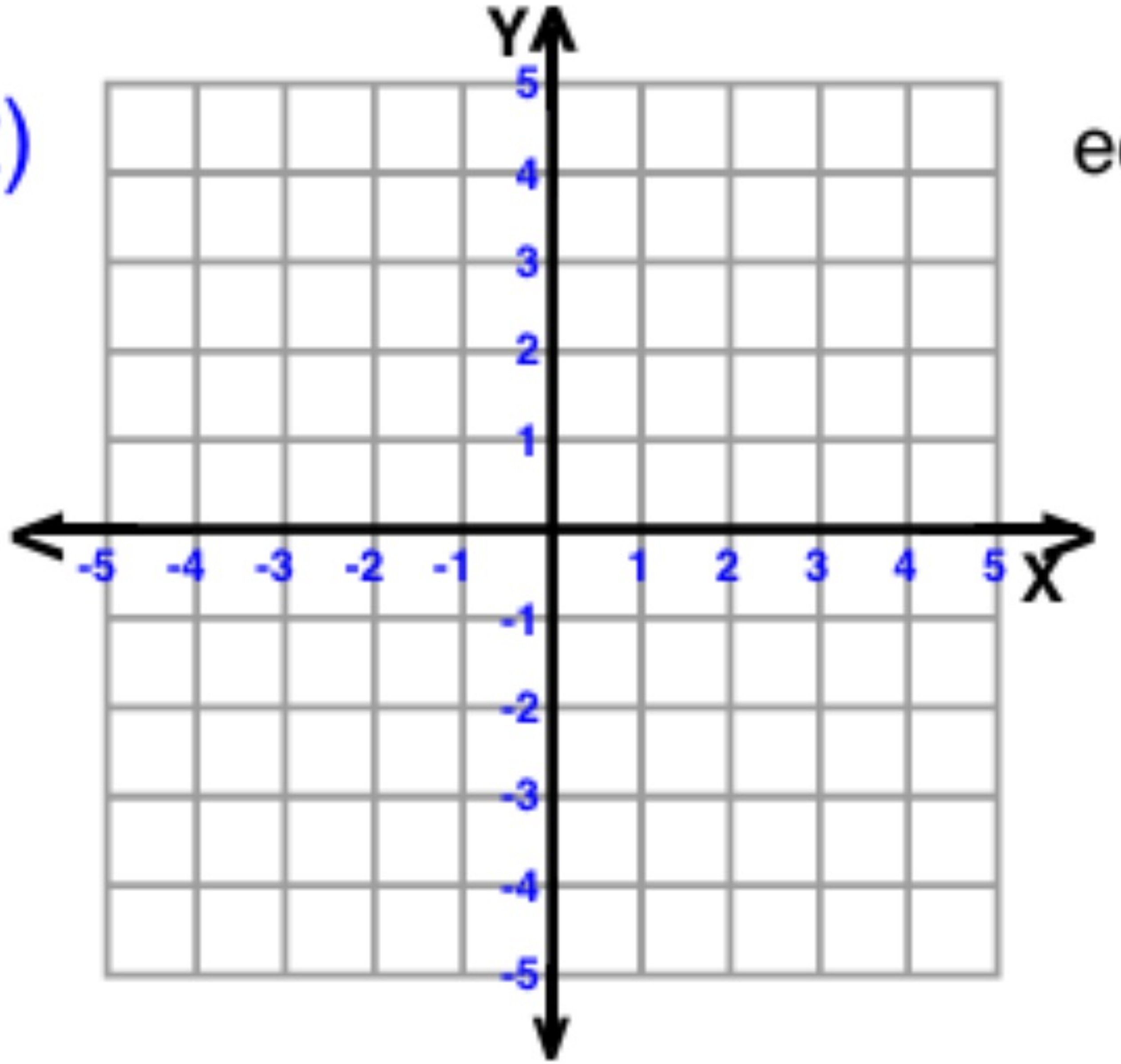

height (yd)
time (s)

Sketch Each Line

1) 

equation $y = -\frac{5}{2}x - 1$

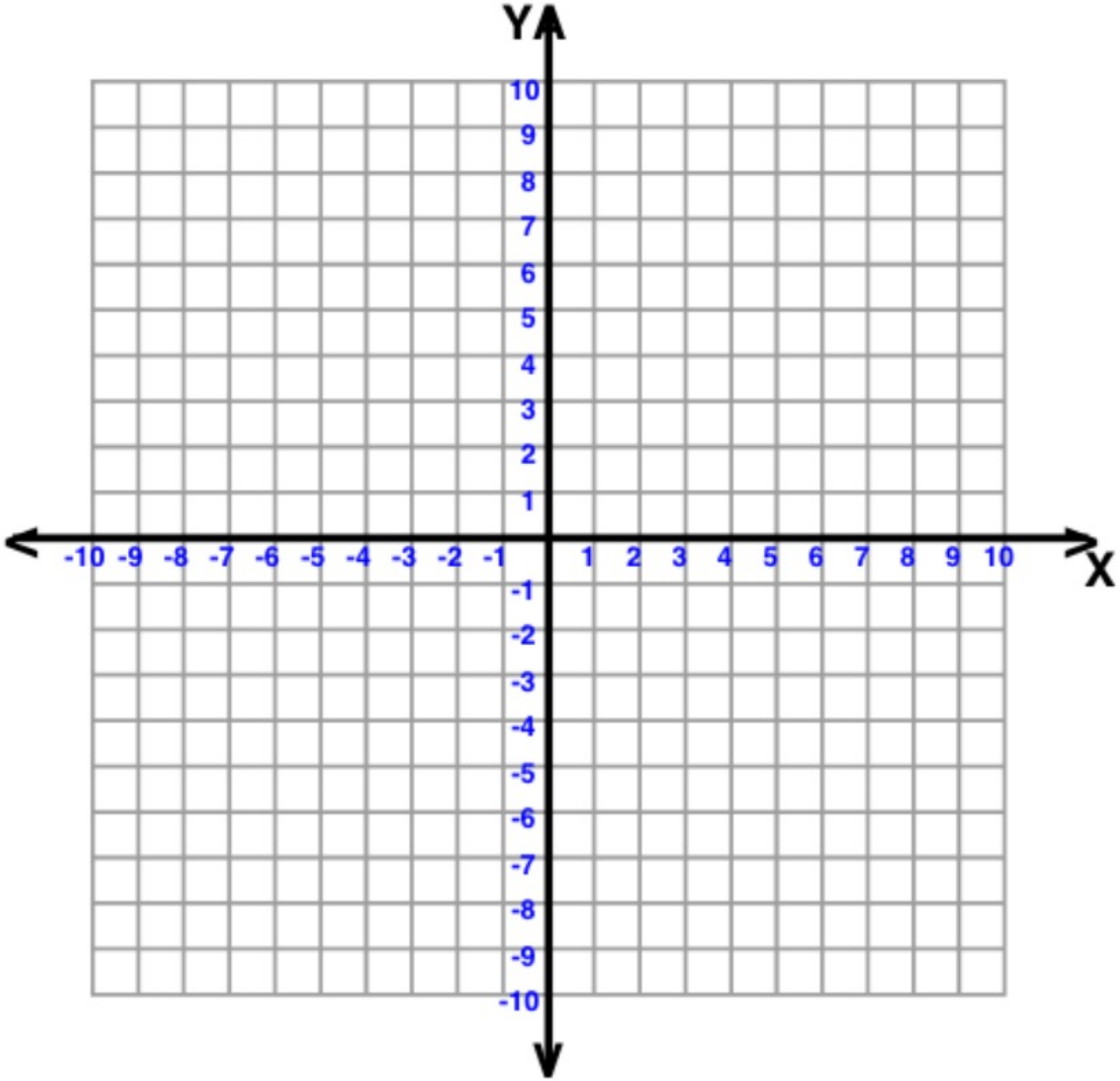
2)



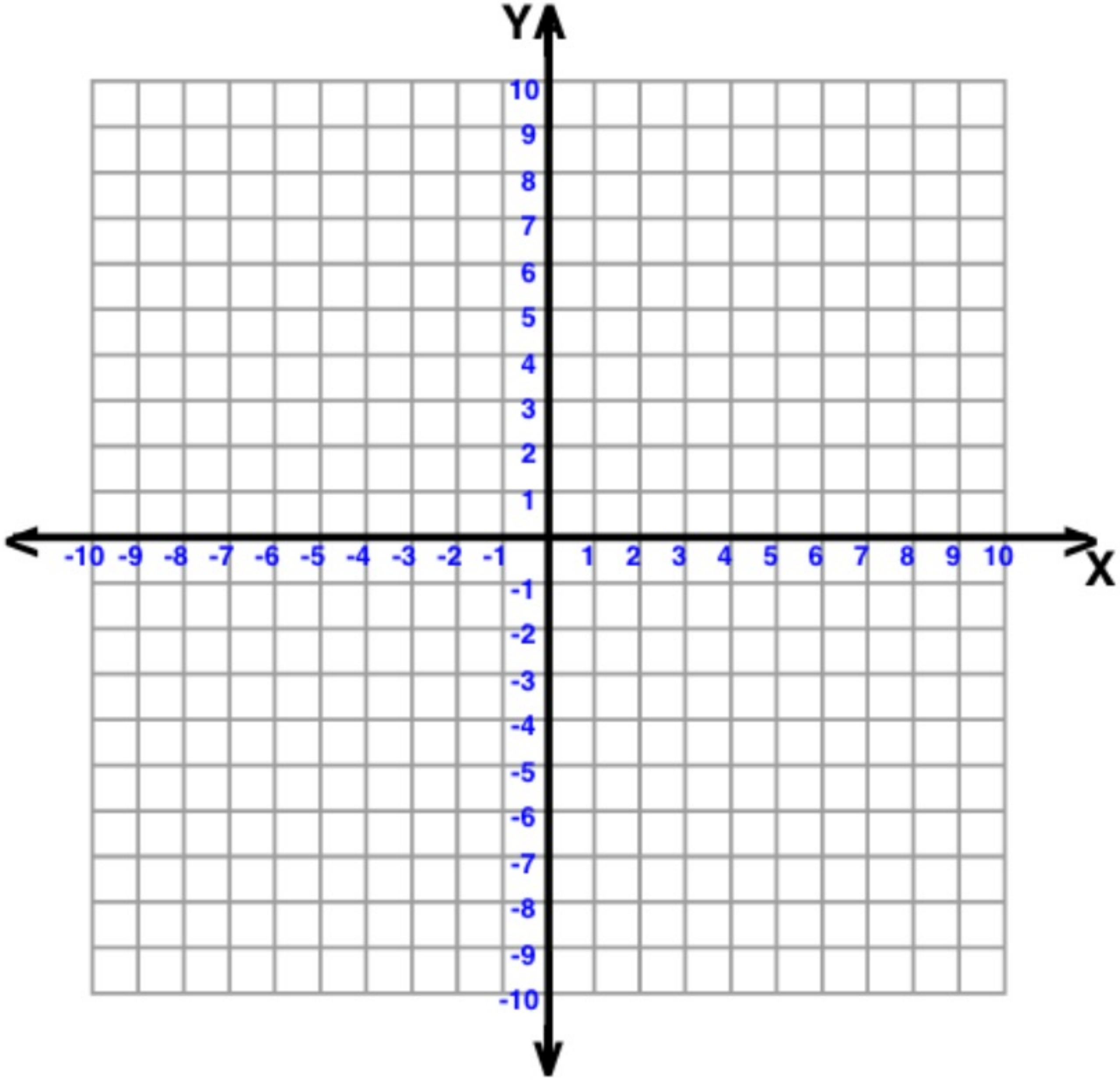
equation $y = \frac{3}{2}x + 3$

Graphing Parabola Equations

1) $y = (x - 2)^2 - 1$



2) $y = 2(x + 2)^2$



$$y = x^2 + 2x - 8$$

- Which of the following pairs of x-values represent where the curve crosses the x-axis?

A. $x = -8$, $x = 8$
B. $x = -4$, $x = 2$
C. $x = -4$, $x = 4$
D. $x = -2$, $x = 4$

$$y = x^2 + 2x - 8$$

- Which of the following values of y represents where the curve crosses the y-axis?

A. $y = -8$
B. $y = -4$
C. $y = 4$
D. $y = 8$

$$y = x^2 + 2x - 8$$

- Which of the following values of x represents where the curve goes through a minimum?

A. $x = -2$
B. $x = -1$
C. $x = 1$
D. $x = 2$

Functions

- A **function** is a relation in which each input has exactly one output. Functions can be represented by sets of ordered pairs in tables, in graphs, algebraically, or by verbal descriptions.
- Two or more functions can be compared based on their slopes of rates of change, intercepts, the locations and values of minimums and maximums, and other features.

Quadratic Function

- The Function $f(x)=ax^2+bx+c$ where a, b, and c are constants and $a \neq 0$ is a ***quadratic function***. Quadratic function in this form is said to be in standard form.

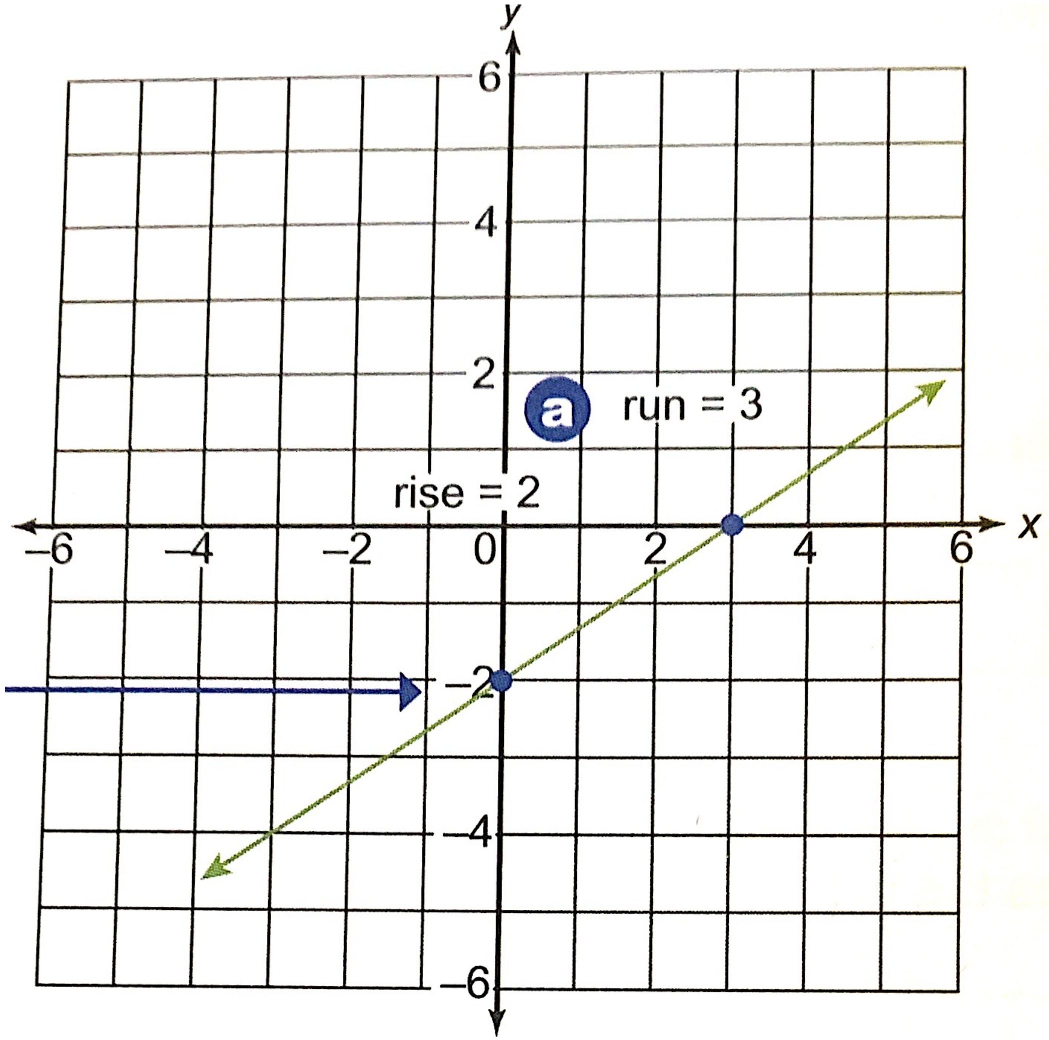
The following are examples of quadratic functions.

$y=x^2$	$a=1$	$b= 0$	$c= 0$
$f(x)=x^2+2x-5$	$a=1$	$b= 2$	$c=-5$
$g(x)=3x^2-4x$	$a=3$	$b=-4$	$c= 0$

TEST-TAKING TIPS

- The rate of change is also known as slope.
- In a graph, the rate of change is the ratio of the vertical change, or rise, to the horizontal change, or run.
- The intercepts of a function are the values of one coordinate when the other coordinate is zero.
- In a graph, look for points that cross the axes. In a table, look at the rows in which one value is 0.
- When functions are presented with different representations (graph/table), you may want to change the representations of one or both functions.

Practice



a

	x	y	
+1	−3	−5	+2
+1	−2	−3	+2
+1	−1	−1	+2
	0 b	1	
	1	3	
	2	5	
	3	7	

1. A function has a rate of change that is greater than the rate of change shown in the graph above and less than the rate of change shown in the table above. Which equation could represent the function?

A. $f(x) = 3x + 2$

B. $f(x) = \frac{1}{2}x - 1$

C. $f(x) = x + 3$

D. $f(x) = \frac{5}{2}x + 2$

DIRECTIONS: Study the table, read each question, and choose the **best** answer.

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

5. Which function has the same x-intercepts as the function represented in the table?

A. $f(x) = \frac{1}{2}x^2 - 2$

B. $f(x) = \frac{1}{2}x^2 + 2$

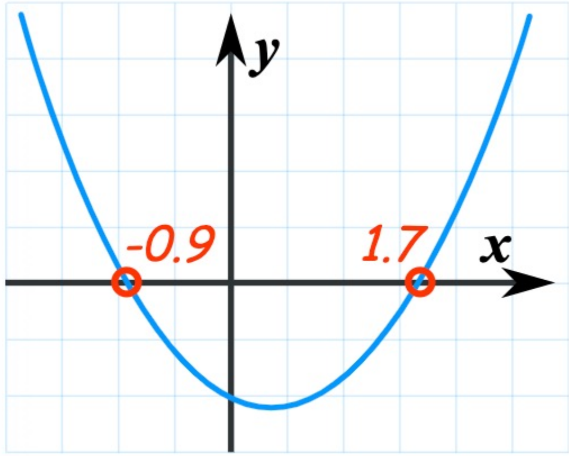
C. $f(x) = 2x^2 - 2$

D. $f(x) = 2x^2 + 2$

How to Solve Quadratic Equations

The "**solutions**" to the Quadratic Equation are where it is **equal to zero**.

They are also called "**roots**", or sometimes "**zeros**"



There are usually 2 solutions (as shown in this graph).

And there are a few different ways to find the solutions:

We can Factor the Quadratic (find what to multiply to make the Quadratic Equation)

Or we can Complete the Square

Or we can use the special **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Just plug in the values of a, b and c, and do the calculations.

We will look at this method in more detail now.

About the Quadratic Formula

Plus/Minus

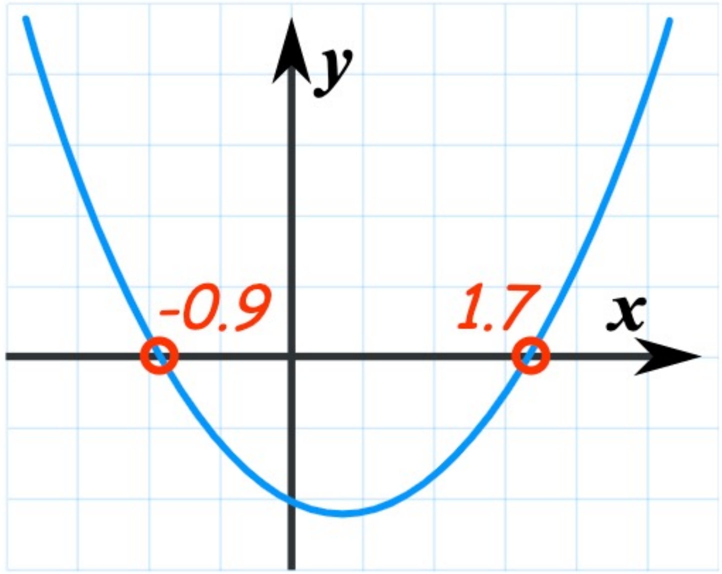
First of all what is that plus/minus thing that looks like $\pm$?

The $\pm$ means there are TWO answers:

$$x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$

Here is an example with two answers:



But it does not always work out like that!

- Imagine if the curve "just touches" the x-axis.
- Or imagine the curve is so **high** it doesn't even cross the x-axis!

This is where the "Discriminant" helps us ...

Discriminant

Do you see $\mathbf{b^2 - 4ac}$ in the formula above? It is called the **Discriminant**, because it can "discriminate" between the possible types of answer:

- when $\mathbf{b^2 - 4ac}$ is positive, we get two Real solutions
- when it is zero we get just ONE real solution (both answers are the same)
- when it is negative we get a pair of Complex solutions

Complex solutions? Let's talk about them after we see how to use the formula.

Example: Solve $5x^2 + 6x + 1 = 0$

Coefficients are: $a = 5,\ b = 6,\ c = 1$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

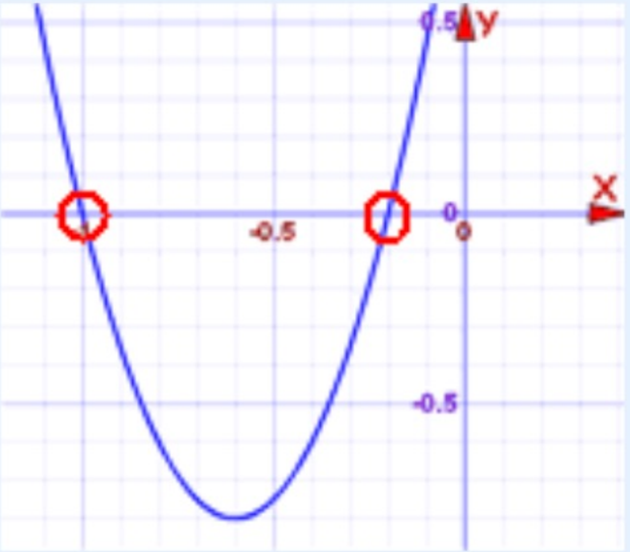
Put in a, b and c: $x = \dfrac{-6 \pm \sqrt{(6^2 - 4\times5\times1)}}{2\times5}$

Solve: $x = \dfrac{-6 \pm \sqrt{(36 - 20)}}{10}$

$$x = \dfrac{-6 \pm \sqrt{(16)}}{10}$$

$$x = \dfrac{-6 \pm 4}{10}$$

$x = -0.2$ **or** -1



Answer: $x = -0.2$ **or** $x = -1$

And we see them on this graph.

Check **-0.2**:

$5\times(\mathbf{-0.2})^2 + 6\times(\mathbf{-0.2}) + 1$

$= 5\times(0.04) + 6\times(-0.2) + 1$

$= 0.2 - 1.2 + 1$

$\mathbf{= 0}$

Check **-1**:

$5\times(\mathbf{-1})^2 + 6\times(\mathbf{-1}) + 1$

$= 5\times(1) + 6\times(-1) + 1$

$= 5 - 6 + 1$

$\mathbf{= 0}$

The Quadratic Formula

Solve each equation with the quadratic formula.

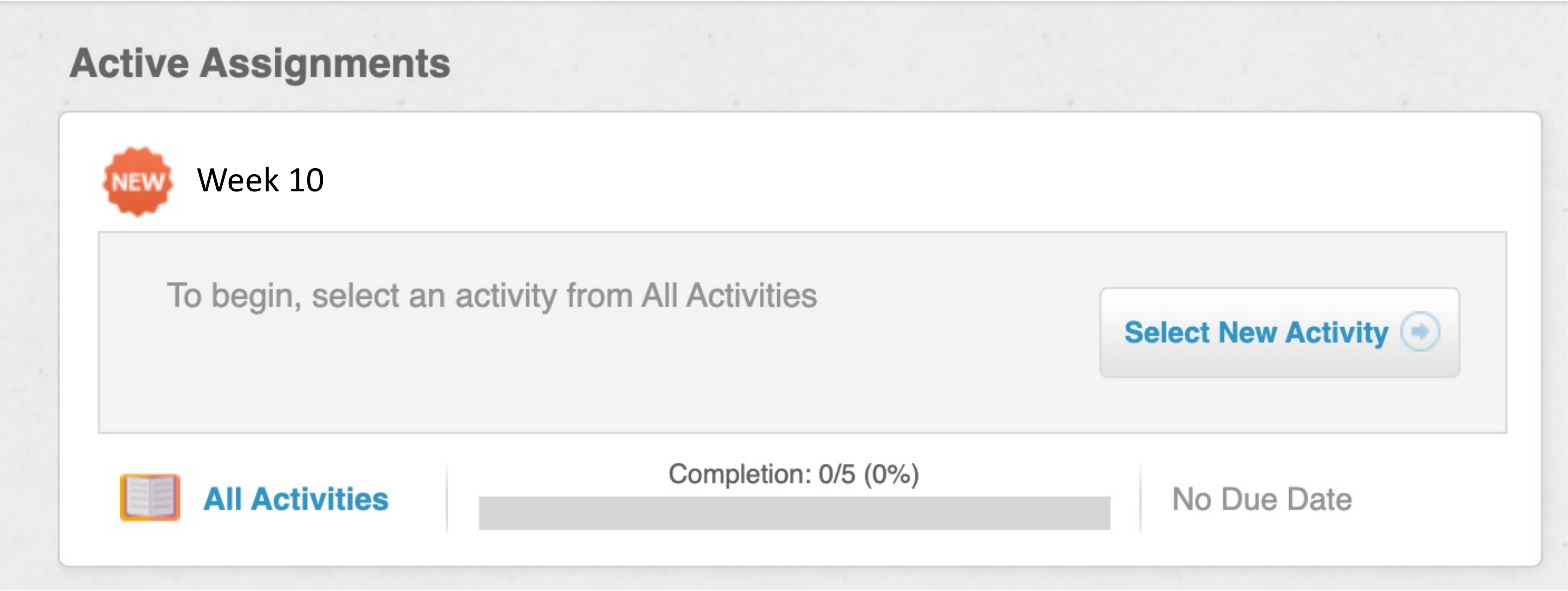
1) $n^2 + 6n - 40 = 0$

2) $6k^2 - 28k - 10 = 0$

6) $n^2 - 16n + 63 = 0$

7) $n^2 - 2n - 48 = 0$

Homework!


Active Assignments
NEW
Week 10
To begin, select an activity from All Activities
Select New Activity
All Activities
Completion: 0/5 (0%)
No Due Date